PUBLIC ENTERPRISES: RESOURCE ALLOCATION

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THESIS

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ABSTRACT

A great deal of controversy has arisen with regard to the practice of marginal cost pricing as a public sector policy to achieve efficient resource allocation. It is important to present the different aspects of this very complex problem together to establish a more clarified aggregate picture.

Marginal cost pricing is faced with three main sources of problems:

1. That of measurement, due to both theoretical and practical difficulties in establishing cost,

2. That of determination of social welfare, a very complex task involving not only economic factors but also those of politics, psychology, sociology and hence not easily justified in a clear-cut manner,

3. That of income redistribution, which generally occurs with the achievement of the optimal social welfare in its ultimate form.

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I. INTRODUCTION

In public economics, the question of how government activities should be managed so as to achieve the efficient use of the limited resources available is one of major interest.

Many economists, especially welfare economists, have examined this problem, trying to find an answer to the question: "What pricing policy should public enterprises adopt?"

Successes as well as frustrations of various degrees have been experienced. It is recognized that the problem is directly associated with the social welfare area, the objective being to maximize social welfare. Thus it is a very complex problem, since social welfare determination is ultimately an ethical judgement in the context of an infinite number of possible conbinations among various factors such as politics, sociology, psychology and economics.

A completely satisfactory answer in this context is not possible and much debate has arisen which tends to confuse the issue. Furthermore, exactly because of its complexity that the available literature consists of articles, each dealing with some of its special aspects only. Hence it is hard to form a consistent aggregate picture of the situation.

This thesis is an attempt to establish this aggregate picture and along the way to pin down the limits and

difficulties associated with the problem of resource allocation faced by public enterprises.

Section II develops the social welfare concept to establish the context in which social welfare is used by economists in their analyses of the public sector behavior.

Section III shows the implication of social welfare on the desired behavior for the economy and the context in which this behavior is possible.

Section IV summarizes the different approaches to the analysis of the public enterprise's behavior.

Section V discusses the implications of the marginal cost pricing behavior, the related difficulties and an assessment of this pricing policy as compared with others.

II. WELFARE ECONOMICS

Scarcity of resources requires an efficient allocation in their use.

What do we mean by making the best use of the resources?

Presumably, resources are being used to satisfy our need, to serve our well-being. Conceptually then, on the national level, efficient allocation of resources would be related to the achievement of maximum social welfare subject to their scarcity.

One has to face, immediately, with the problem of what constitutes social welfare and how different states of welfare can be ranked.

On this question, Mrs. Ruggles (Ref. 1) gave us a rather useful review on the development of welfare economics from the so called "old" to "new" welfare views.

Basically, it is recognized that social welfare is some amalgam of the welfares of the constituent members of the population, who in the words of Mishan (Ref. 2) are assumed to be "rational" and "responsible" beings.

Rational in two senses:

-That the choices made by each individual in any situation are consistent with his other choices. (Thus an individual "welfare" is reflected through his choice when he has the freedom to do so).

-That the well-being of the individual depends only on his own real income and not at all on those of the others.

Responsible in that each individual is taken to be the best judge of his own wants.

Clearly, both senses of "rational" are simplifications which may or may not be true, while individual responsibility is an ethical judgement (unless the individual always knows with certainty what is best for him).

Note also that in specifying his choice, the individual needs only rank his preferences, thus individual welfare (or utility) is ordinal in nature.

How are individual welfares taken into account in the realm of social welfare?

Earlier welfare economists, while recognizing the problem of making interpersonal comparisons of utilities, argued that it is necessary to do so if economic analysis is to be significant. Bentham went as far as to propose the sum total of happiness as a measure of social welfare which involved treating everybody equally and also that utilities are in fact additive (thus utilities have to be cardinal in nature). Wicksell argued that a rich man carries his consumption so far that the marginal utility of the last unit consumed is little or nothing to him while the poor man must discontinue his consumption on practically every commodity at a point where they still represent for him a very high marginal utility. Thus an exchange of income between the rich man and the poor man might lead to a much

greater total sum of utility among them. This statement clearly implies an interpersonal comparison of utilities had been made. Samuelson's social welfare function is a mathematical statement of the same thing, which would describe a social indifference map over the individual welfares, an analogy to an individual indifference map over the commodities consumed. Social welfare ranking, in this ultimate meaning, is so specific as to contain a high degree of arbitrariness in the sense that there is no definite and unique way of expressing the social welfare function in the context of the total economy and the individuals in it.

It was Arrow (Ref. 3) who tackled the proposition that: "If we exclude universal social rankings based upon interpersonal comparisons of utility and rankings of the dictatorial variety, is it possible to construct a universal social ranking rule that is consistent with the fundamental ethical postulate while taking into account of individual social rankings?"

To this proposition Arrow has provided a very interesting proof in his famous Theorem of Possibility.

Essentially, he set out by characterising that the universal social ranking rule possesses the three fundamental properties of ordering - namely completeness, reflexivity and transitivity -, that there be no imposed or dictated preference, that individual preference should be taken into account but no interpersonal comparison is allowed. Can such a rule exist? From these assumed properties Arrow

demonstrated that the resulting social ranking would reflect the preference of a group, called the decisive group, that any decisive group will contain an even smaller one which is decisive. The argument thus lead to a single individual whose preference will be reflected by the postulated social ranking rule, which then implies dictatorship, which is a contradiction.

Hence Arrow's Theorem of Possibility asserts that if rules based upon interpersonal comparisons of utility are excluded, as are dictatorship rules, then there is no well-behaved universal social welfare ranking rule (i.e. satisfying the assumed properties).

Implicit in the condition that no interpersonal comparisons be allowed is the fact that individual intensity of preference will not count. Hilbreth objected to this less-appealing aspect of the rule, but in the context of the general nature specified by Arrow for the social ranking rule, this is inevitable.

It is thus recognized that one can only hope to reduce the ambiguity in the social welfare ranking rule by using those which are weaker and less specific. One such type is the well-known Pareto ranking rule, also referred to as a partial ranking as contrasted with the complete ranking discussed in connection with the Arrow possibility theorem.

Essentially the Pareto ranking rule states that in going from state Z^1 to Z^2, the social welfare is increased if some individuals are made better-off and none worse-off.

Diagrammatically, the Pareto ranking rule can be presented as in Figure 1, where OAB represents the set of possible states of welfare distribution for a two-individual society (this assumption is made only for ease of graphical presentation). Any state on the curve AB is considered Pareto optimum since there is no way of increasing U^1 without decreasing U^2 and vice versa. No two Pareto optimum states are comparable in the Pareto sense (unless one is prepared to make interpersonal comparisons of utility, i.e. making the ethical judgement in the sense of Samuelson's social welfare function). Any state inside the shaded area PRS is considered as an improvement of P. Now consider C and P; nothing can be said about them in the Paretian sense except that P, being a Paretian non-optimum, can be improved upon. They are non-comparable except in the ultimate sense of the social welfare function type of judgement in which case anything can happen - C may be better or worse than P.

This Pareto ranking rule, although greatly reducing the amount of ambiguity associated with ethical judgement, suffers in that it is infrequent that real situations occur in the way that this rule can be used, i.e. some are better-off and none worse-off. More likely there would be some gainers and some losers. Attempts have been made to find other criteria for social ranking that would allow judgement to be made on these situations. The one such well-known criterion is the Hicks-Kaldor compensation principle.

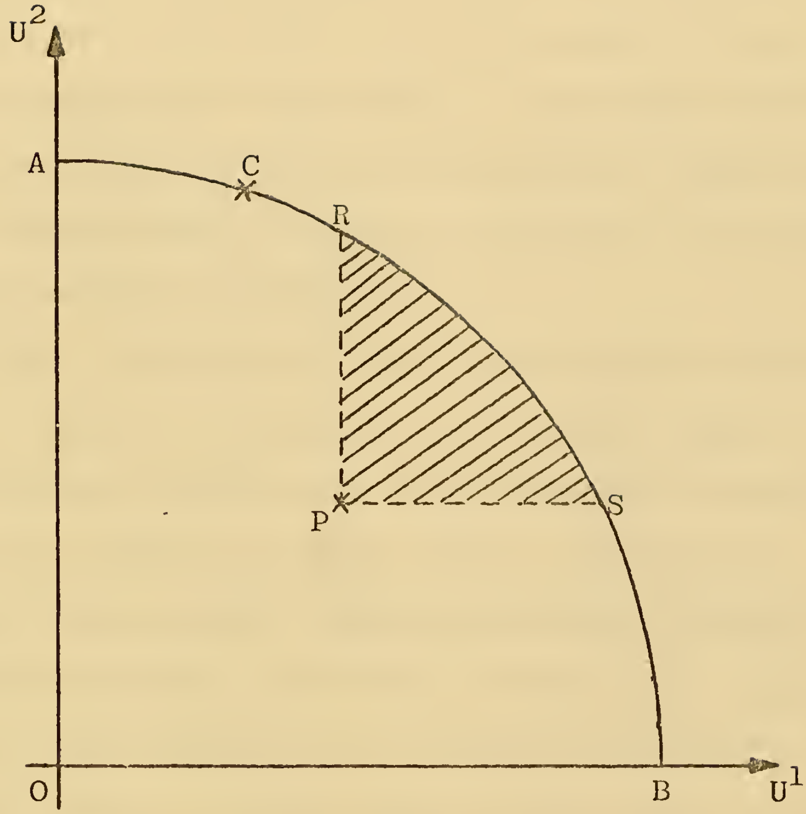

Figure 1

According to this principle a change should be made if the amount of "income" that the gainers are willing to pay for the change to be carried out exceeds the amount that the losers would be willing to receive to accept the change. In other words, if the change could be made so that someone is better-off and none worse-off. In this sense, the new position is a potential improvement. Thus Hicks-Kaldor avoided the necessity of making interpersonal comparisons of utility by contemplating a transfer of income and comparing the resulting welfare distribution.

Kaldor (Ref. 4) maintained that the transfer need not be made since if this is the case, we are brought back to the situation where the Pareto rule could be applied readily. The compensation principle would then be superfluous.

It may be objected that suppose the change results in one rich person becoming very rich and many poor people becoming very poor, however if a transfer of income is possible then everyone could be made either better off or not worse-off, the compensation principle would approve the change even though the transfer is not made - a decision which does not appear reasonable. This objection can be justifiable only if interpersonal comparisons of utility are made, namely those between the rich and the poor. But this is exactly the type of action that the compensation principle or the Pareto ranking rule do not want to include, and in the light of this ultimate judgement, described by a social welfare function, even a Pareto move does not guarantee

a more favorable solution than a non-Pareto move. Figure 2 shows that, with the assumed community indifference map, the Paretian move PB does not attain as high a welfare state as does the non-Paretian move PA.

The real trouble with the compensation principle was pointed out by Scitovsky (Ref. 5) who showed that the compensation principle has introduced sufficient ambiguity that the use of it sometimes can lead to a contradiction. To prevent this from happening, he suggested that a backward test should also be satisfied. Thus the change should be made only if:

1. The amount that the gainers are willing to pay for the change to be carried out is more than sufficient to compensate the losers.

2. The would be losers cannot "bribe" the would be gainers from not wanting the change.

Scitovsky's findings essentially can be illustrated by Figure 3a and Figure 3b where, for simplicity of the graphical presentation, only a two-commodity two-person economy is considered.

Figure 3a represents two aggregate economy states and their corresponding utility frontiers OQ and OP, which are redrawn in Figure 3b as QQ and PP respectively. Suppose the economy is originally in the Q aggregate state with the welfare distribution at A. Now a change is possible which would bring the economy to the P aggregate state with the welfare distribution at B. The forward test would result

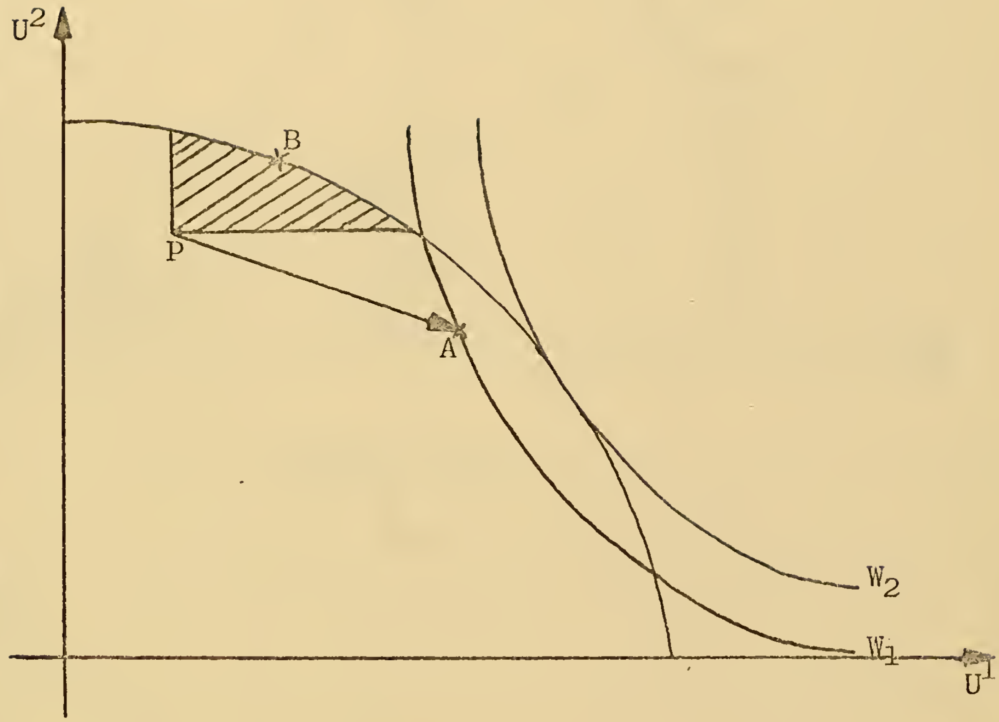

Figure 2

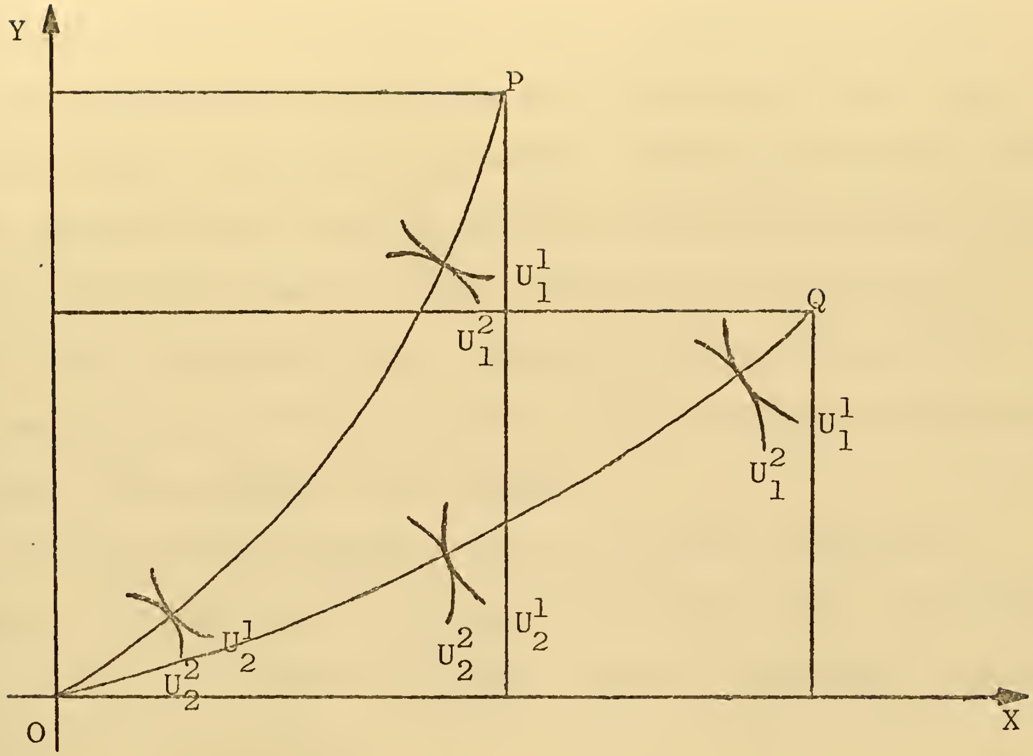

Figure 3.a

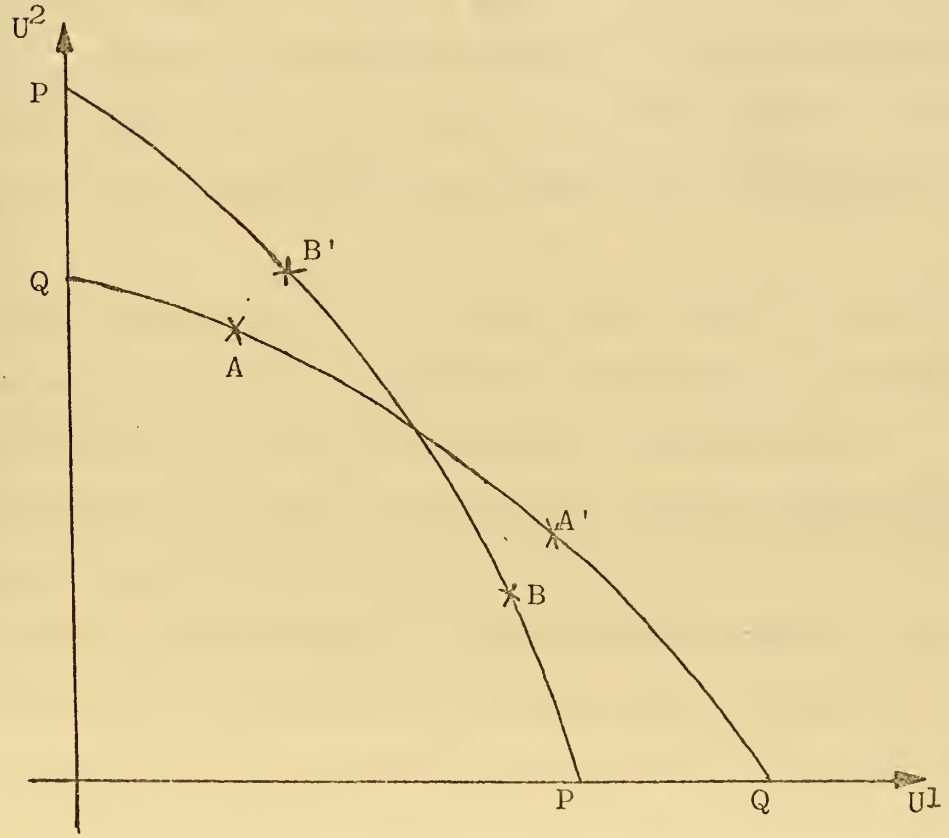

Figure 3.b

in the new welfare distribution at B' superior to A and thus the compensation principle would recommend that the change be made. However, Scitovsky rightly pointed out that it was possible that the "would be" losers could bribe the "would be" gainers and brought the welfare distribution from A to A', superior to B. Thus the criterion used by the compensation principle could lead to an ambiguous situation when the backward test failed.

The forward and backward tests form the Scitovsky's criteria. Unfortunately, it may be possible that even when the Scitovsky's criteria are met, its use can still lead us to logical inconsistency.

Figure 4 illustrates the point where it can be seen that, by the Scitovsky's criteria, A_2 is superior to A_1, A_3 to A_2, A_4 to A_3. By the transitive property of logic we would have A_4 being superior to A_1. However the Scitovsky's rule would say that A_1 is superior to A_4, hence the inconsistency of logic.

Thus the Scitovsky's criteria and, for that matter, the compensation principle cannot be used alone as a guide to policy without the risk of possible contradiction.

Furthermore, even in the sense of being potentially superior the Scitovsky's rule can still lead to ambiguity.

Figure 5 shows that B is potentially superior to A under the Scitovsky's rule, but the associated aggregate state of the economy P is not unambiguously potentially superior to Q since the Scitovsky's rule would say the reverse if B' and A' are to be considered instead.

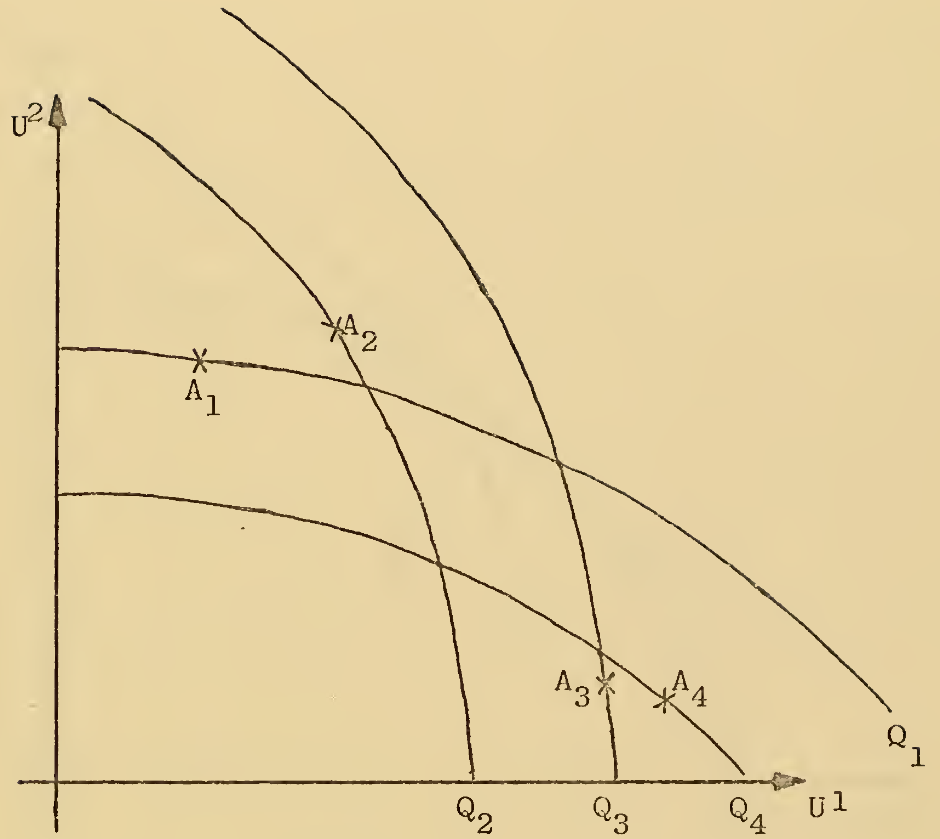

Figure 4

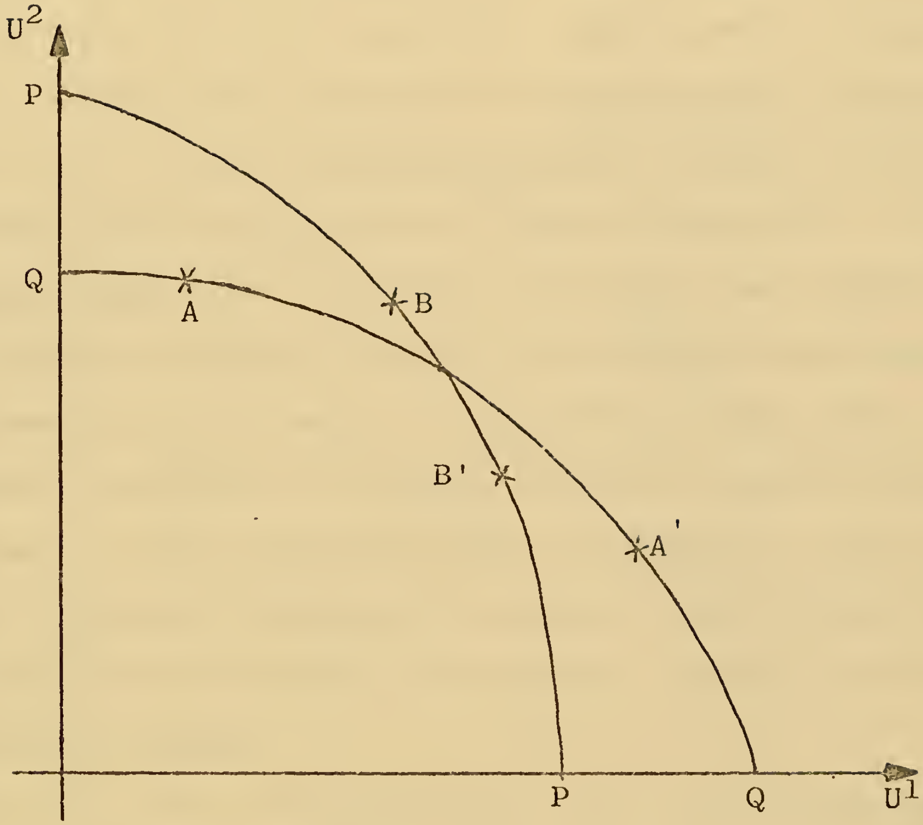

Figure 5

To remove this possible ambiguity, Samuelson (Ref. 6) proposed that the change, to be superior, should be such that the utility possibility curve, derived from the resulting aggregate state of the economy, lies everywhere outside that derived from the initial aggregate state of the economy. Figure 6 illustrates the situation. Equivalently, Samuelson's criteria is such that the change is unambiguously potentially superior only if the resulting aggregate economy has more in some commodities and none less in other commodities. It is only potentially superior in that one cannot say anything about A and B in Figure 6 due to the distributional question.

In summary, with respect to providing a relatively unambiguous criterion for judging the social welfare which can be used as some guidance toward developing a policy for efficient resourse allocation, it appears that we will have to be satisfied with Pareto or Samuelson criteria whenever the situation allows.

The Pareto rule, when possible, will offer unambiguously actual improvement, whereas the Samuelson rule will offer only unambiguously potential improvement. The trade-off is that a wider class of economy states can be judged using the latter criterion.

These criteria exclude interpersonal comparisons of utility, the role of politics, psychology, sociology, etc. These factors will eventually have to be taken into account by those charged with responsibility over the social welfare, and may result in a partial welfare indifference map. In

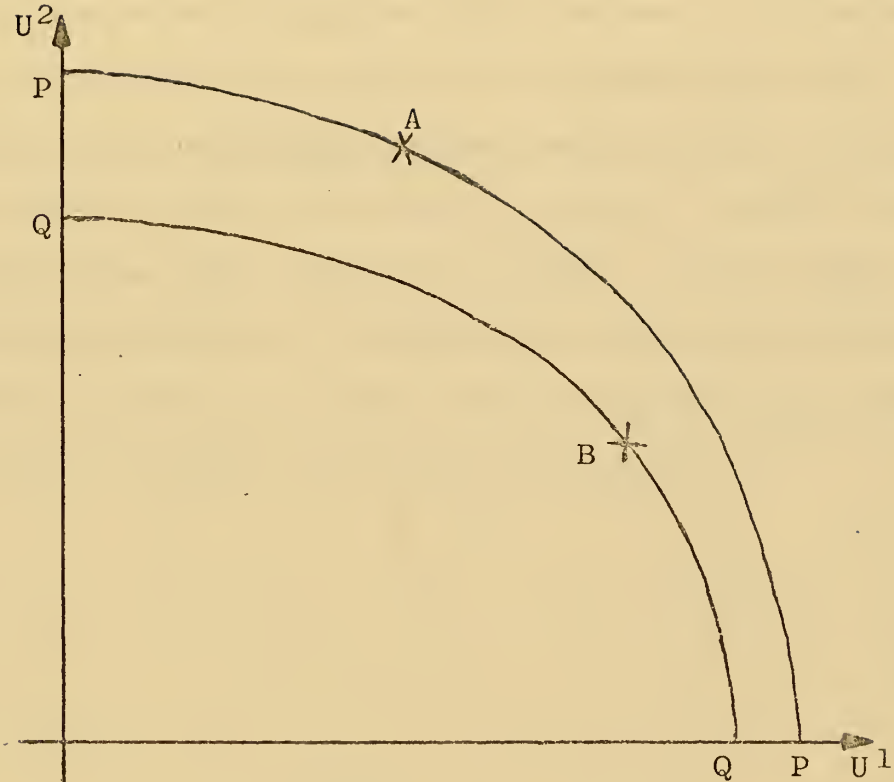

Figure 6

this ultimate judgement, the fact that Paretian optimum or Samuelsonian optimum conditions have been achieved in no way guarantees a favorable verdict.

However, this is not to say that Pareto or Samuelson criteria are no better than any arbitrary criteria. It is seen from the above discussion that the use of these two criteria will lead to the states of economy where one commodity or one individual welfare would attain the highest possible level, given the levels of others. Mishan called these states the lower levels of optima. In his words, to sum up the discussion, "...Though the top of the edifice, the complete optimum, has been shown to be illusory, the lower levels of optima are fairly substantial..."

III. PARETO OPTIMUM CONDITIONS

The discussion on welfare economics leads us to the Samuelson and Pareto criteria which imply that, for the study of optimum conditions, we have to look at "efficient" production and exchange.

Millward's (Ref. 7) approach has been used in this presentation.

Assume an economy of n goods; the input goods will be negative and the output goods positive. There are v producers, each of which has a production function of the type

$$f^a(x_1^a, \ldots, x_n^a) = 0 . \qquad a = 1, \ldots, v$$

where x_j^a is the amount of the j^{th} good produced or used by producer a. The production functions are continuously differentiable.

Then efficient production can be mathematically formalized as:

$$\text{Max } X_1$$

subject to:

$$\sum_{a=1}^{v} x_j^a = X_j^* . \qquad j = 2, \ldots, n$$

$$f^a(x_1^a, \ldots, x_n^a) = 0 . \qquad a = 1, \ldots, v$$

where X_j^* is the given aggregate level of the j^{th} good produced or consumed in the economy.

Assuming the second order conditions to be satisfied, the first order conditions would lead to:

$$\frac{F_j^a}{F_k^a} = \frac{F_j^b}{F_k^b} \quad (1) \qquad a, b = 1, \ldots, v \quad j, k = 1, \ldots, n$$

where $$F_j^a = \frac{\partial f^a}{\partial x_j^a}$$

-If k is input and j output, then (1) says that the marginal product of k in terms of j should be the same for all firms using k to produce j. Clearly if this does not hold then X_j can be increased by reallocating X_k among firms.

-If both are inputs then (1) says that the relative marginal productivity of any two given factors should be the same for every firm.

-If both are outputs then (1) says that the marginal cost of j in terms of k should be the same for all firms.

The first order conditions, together with the constraints, can be used to solve for X_1 in terms of the assumed levels of the X_j^*'s. If the assumed levels of X_j^*'s are changed, then X_1 would take different values. The result would be described by a social transformation function denoted as:

$$F(X_1, \ldots, X_n) = 0 \quad . (2)$$

If the inputs are fixed (e.g. the time period considered is short enough so that resources can be considered as inelastic) then (2) describes a production frontier surface in the output space.

In the exchange problem, we assume there are m individuals, each having a utility function of the form:

$$U^i = U^i(x_1^i, \ldots, x_n^i)$$

where x_j^i is the amount of the j^{th} good consumed (if positive) or supplied (if negative) by the i^{th} individual. The utility functions are assumed continuously differentiable.

Then efficient exchange can be mathematically formalized as:

$$\text{Max } U^1(x_1^1, \ldots, x_n^1)$$

subject to:

$$\sum_{i=1}^{m} x_j^i = X_j^* \qquad j = 1, \ldots, n$$

$$U^i(x_1^i, \ldots, x_n^i) = U^{*i} \qquad i = 1, \ldots, m$$

Again assuming the second order conditions are satisfied, the first order conditions lead to:

$$\frac{U_j^i}{U_k^i} = \frac{U_j^s}{U_k^s} \qquad (3) \qquad \begin{matrix} i, s = 1, \ldots, m \\ j, k = 1, \ldots, n \end{matrix}$$

where

$$U_j^i = \frac{\partial U^i}{\partial x_j^i}$$

-If j, k are both consumed goods then (3) says that the marginal subjective rate of substitution between j and k (in consumption) is the same for all individuals.

-If j, k are both supplied goods then (3) says that the marginal rate of substitution between factors supplied is the same for all individuals.

-If j is supplied while k is consumed then (3) says that the willingness to supply j in exchange for the consumption of k is the same for everyone at the margin.

Similarly, the first order conditions, together with the constraints, will give us a relation:

$$H(U^1, \ldots, U^m) = 0. \quad (4)$$

which describes possible utilities distribution as related to aggregates of outputs and inputs. In the utility space it represents a utility-possibility surface.

The efficient production and exchange conditions can be related to each other by formulating the following problem:

$$\text{Max } U^1(x_1^1, \ldots, x_n^1)$$

subject to:

$$F(X_1, \ldots, X_n) = 0.$$

$$U^i(x_1^i, \ldots, x_n^i) = U^{*i} \qquad i = 2, \ldots, m$$

$$\sum_{i=1}^{m} x_j^i = X_j \qquad j = 1, \ldots, n$$

Again the first order conditions lead to:

$$\frac{U_j^i}{U_k^i} = \frac{U_j^s}{U_k^s} = \frac{F_j}{F_k} \qquad (5) \qquad \begin{matrix} i,s = 1, \ldots, m \\ j,k = 1, \ldots, n \end{matrix}$$

where

$$U_j^i = \frac{\partial U^i}{\partial x_j^i}, \quad F_j = \frac{\partial F}{\partial x_j^i} = \frac{\partial F}{\partial X_j}$$

-If j and k are outputs then (5) says that the rate at which it is technically possible to transform k into j should be equal to the common individual relative evaluation of the two commodities.

-If j is output and k is input, then (5) says that the rate at which individual weights the loss (disutility) in providing factor services against the benefit of the product of these services should be equal to the rate at which it is possible to transform factor services into commodities.

-If j and k are both inputs, then (5) says that the rate at which factor suppliers are prepared to switch their supplies should be equal to the rate at which it is technically possible to do so.

The first order conditions, together with the constraints may make possible the relation

$$G(U^1, \ldots, U^m) = 0.$$

which gives us the various welfare distributions corresponding to the social transform function

$$F(X_1, \ldots, X_n) = 0.$$

The function G will represent, in the utility space, a surface called "the grand utility possibility surface".

For a unique solution to be possible, the social ranking rule must be so specific as to provide us with a social welfare function $W(U^1, \cdots, U^m)$. Then the solution for the "grand design" can be found by solving the problem:

$$\text{Max } W(U^1, \ldots, U^m)$$

subject to:

$$F(X_1, \ldots, X_n) = 0.$$

where:

$$\sum_{i=1}^{m} x_j^i = X_j \qquad j = 1, \ldots, n$$

$$U^i(x_1^i, \ldots, x_n^i) = U^i \qquad i = 1, \ldots, m$$

The first order conditions will lead to:

$$W_i U_j^i = W_s U_j^s \quad (6) \qquad i, s = 1, \ldots, m \quad j = 1, \ldots, n$$

where $W_i = \dfrac{\partial W}{\partial U^i}$, $U_j^i = \dfrac{\partial U^i}{\partial x_j^i}$

and that $\dfrac{U_j^i}{U_k^i} = \dfrac{U_j^s}{U_k^s} = \dfrac{F_j}{F_k}$

which is the same as (5).

Thus Pareto optimum conditions are necessary for the "grand optimality".

Bator (Ref. 8) expounded the above points graphically in a very simple and effective manner. To make his diagrammatic presentation possible he has to limit himself to the cases of two inputs (L and D) inelastically supplied, two outputs (A and N), and two individuals. He also made a host of other assumptions, some of them strong, to ensure convexity, no externalities, smooth continuity.

Here we need only show the diagrams and relate them to what has been shown earlier.

Figure 7 - the curves labeled A_1 and A_2 represent different output levels for A. Similarly do N_1 and N_2 for N. O_aO_n represents the production possibility function $F(X_1, \cdots, X_n)$ in the input space for this particular case.

Figure 8 - FF represents $F(X_1, \cdots X_n)$ in the output space; OP the utility possibility curve in the commodity space as related to production point P.

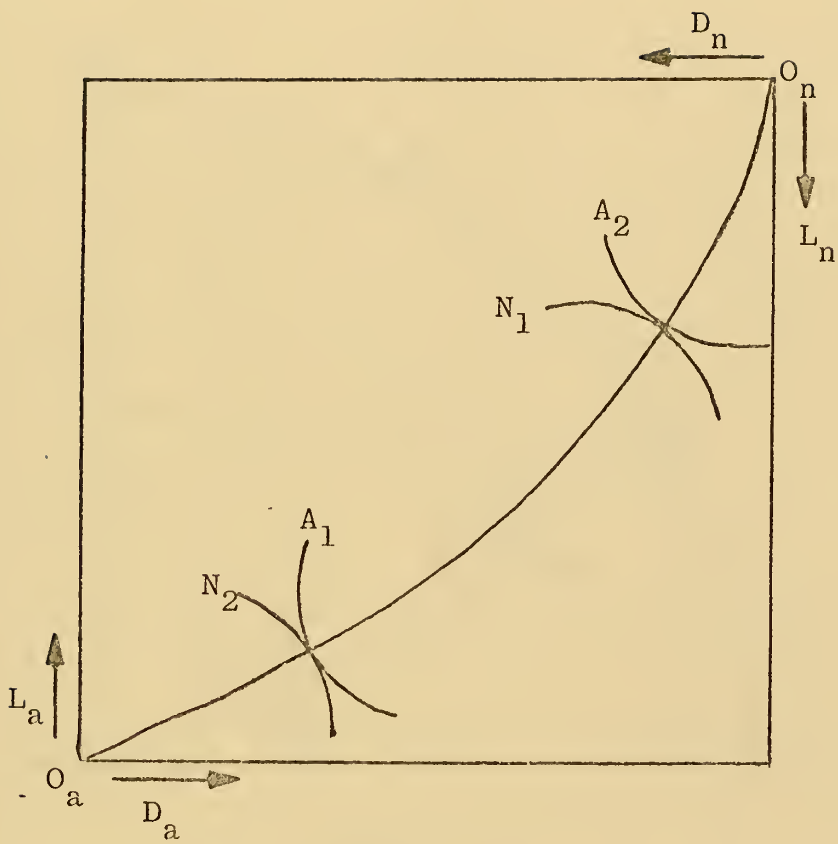

Figure 7

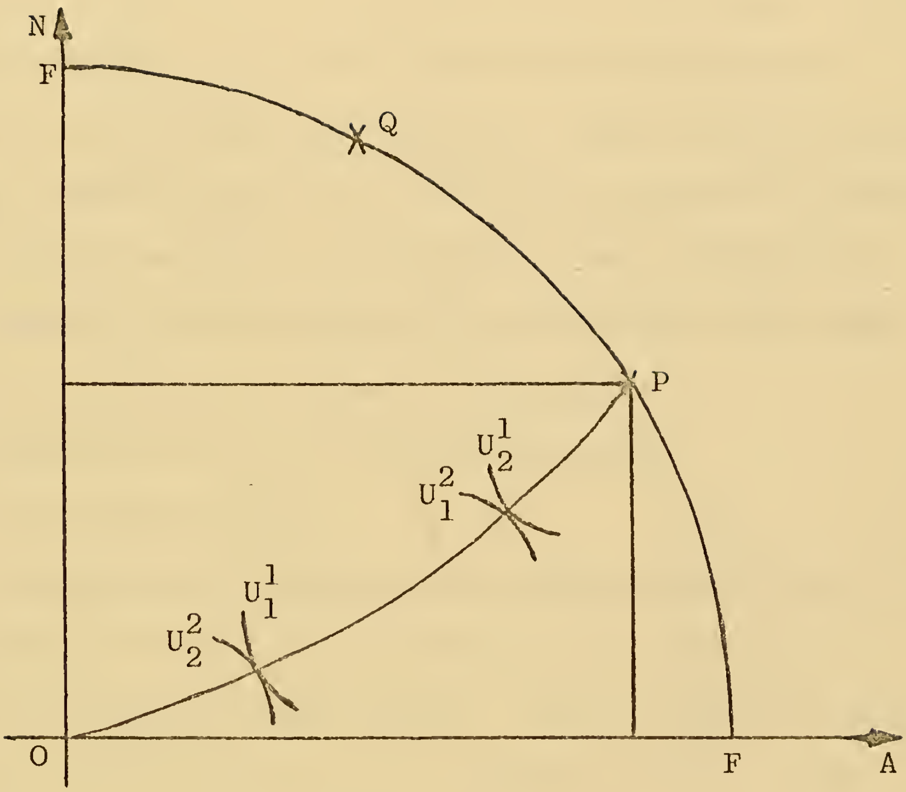

Figure 8

Figure 9 - PP represents the possibility curve $H(U^1, \cdots U^m)$ related to production point P in the utility space; ΩΩ is the possibility curve related to the production point Q in Figure 8.

Figure 10 - GG represents the grand utility possibility surface $G(U^1, \cdots U^m)$ related to the production possibility curve FF in Figure 8; W_1, W_2 represents different levels of social welfare; S is the grand optimum solution.

Now it is important to emphasize that the above results have been obtained under the following necessary assumptions:

- Convex behavior in production and consumption
- Smoothly differentiable curves which also imply perfect divisibility of inputs and outputs
- Tangency occurs at internal points
- No externalities

a. The internal tangency solution implies that, in the case of production, at the optimum point each input is required to produce every output. This is not necessarily the case if the solution is at a corner point. This is illustrated in Figure 11a and Figure 11b; the latter shows that:

$$\left(\frac{\text{Marginal productivity of L}}{\text{Marginal productivity of D}}\right)_A < \left(\frac{\text{Marginal productivity of L}}{\text{Marginal productivity of D}}\right)_N$$

A similar situation can happen in the exchange problem, as is shown by Arrow, when a commodity is not necessarily consumed by everyone. Figure 12 illustrates the situation.

b. When externalities exist, the first order conditions may no longer involve just a particular factor service, a

Figure 9

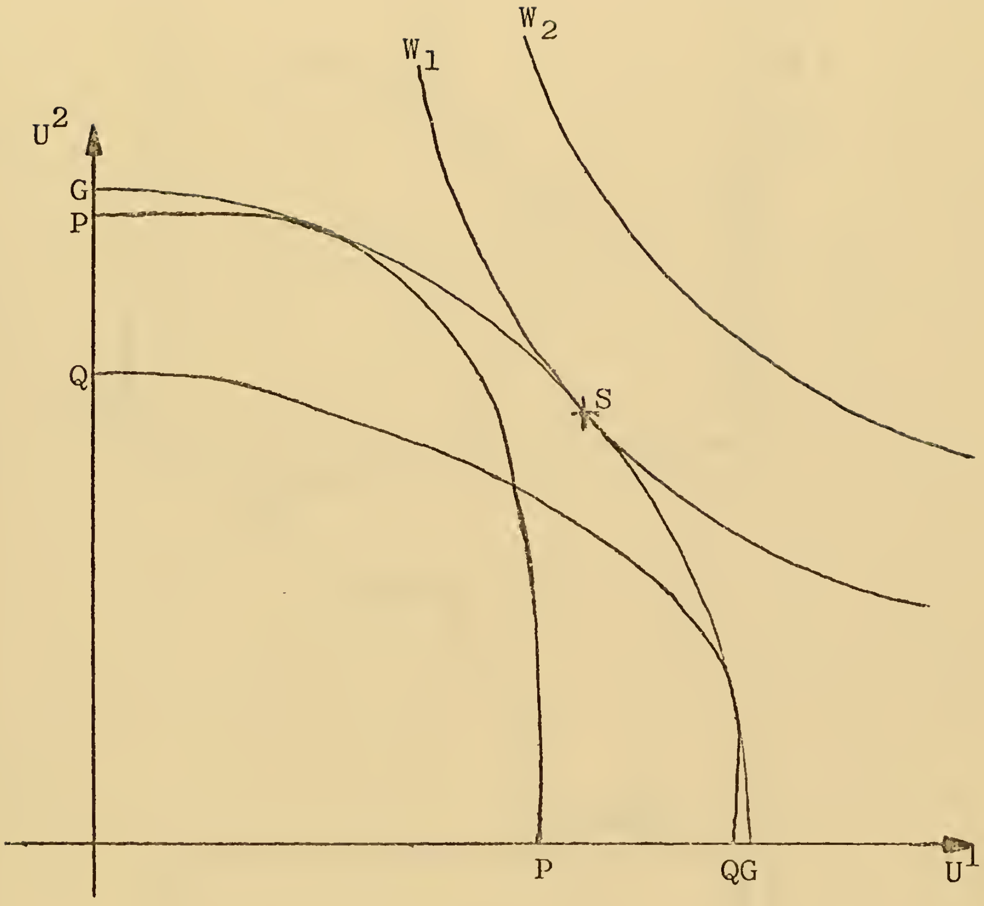

Figure 10

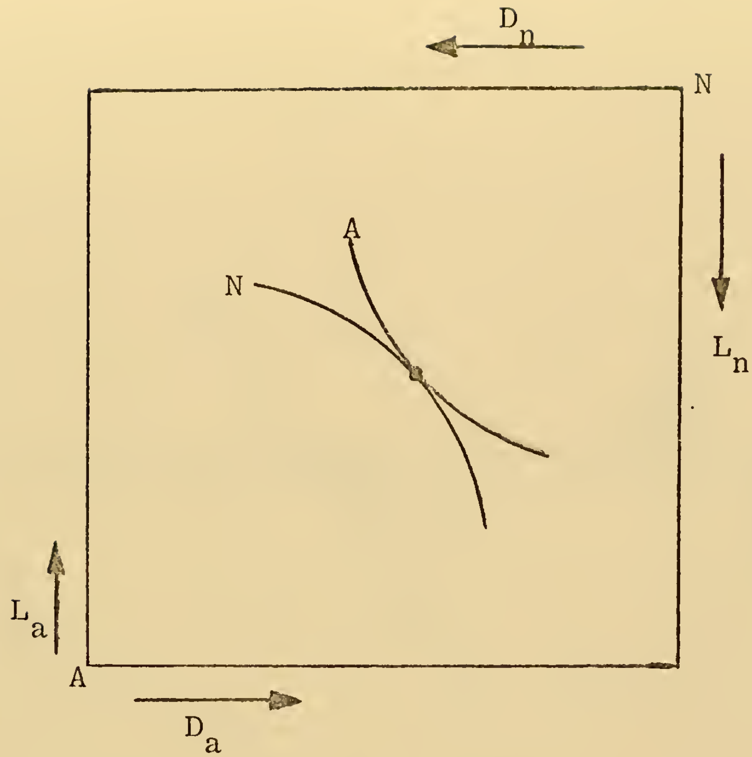

Figure 11.a

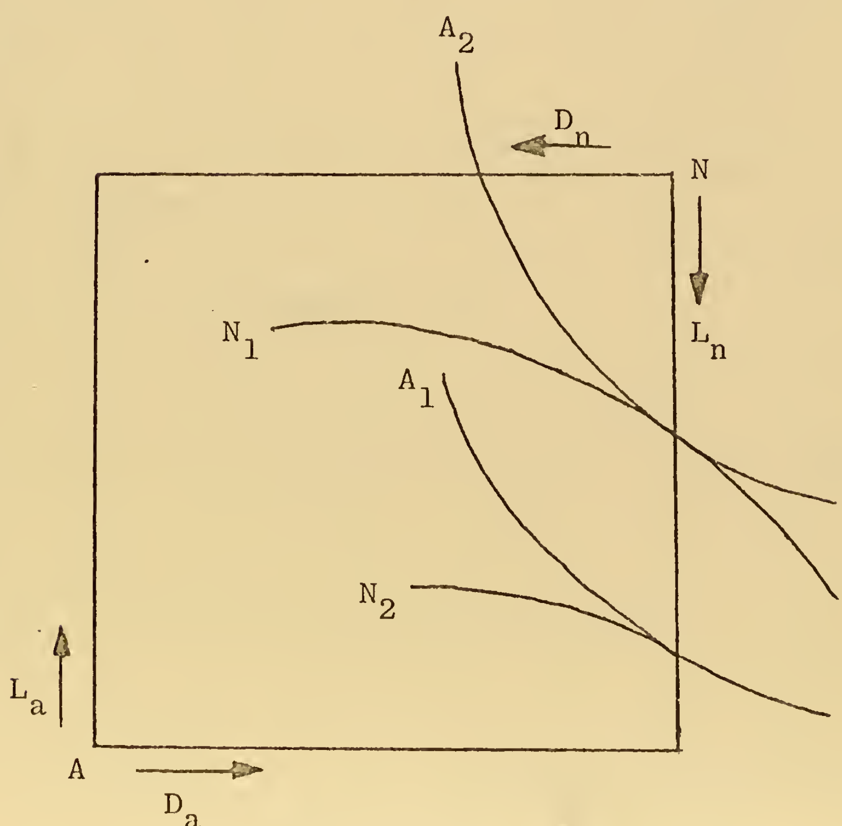

Figure 11.b

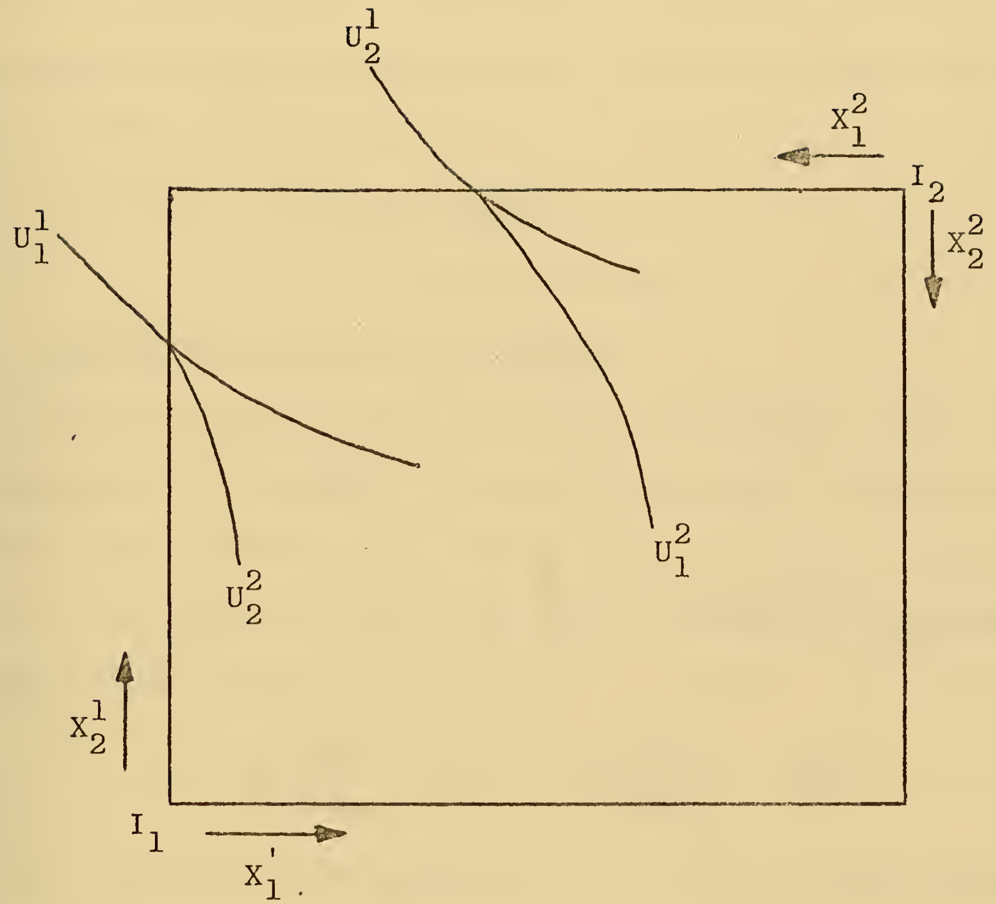

Figure 12

particular output or a particular individual. Instead they reflect some "hidden" inputs or outputs, the benefits or costs of which are not easily appropriated by market institutions. A simple example was given by Meade who assumed an economy of one single input labor resource $\bar{L}$, with two homogeneous and divisible goods: apples (A) and honey (H) with production function

$$A = A(L_A)$$

$$H = H(L_H, A(L_A))$$

Perfect competition is assumed.

Then the apple growers and honey producers acting independently through the competitive market would be producing their products at such a level so that the labor wage is W satisfying (let P_A, P_H be the prices of apples and honey respectively):

$$P_A \frac{\partial A}{\partial L_A} = \omega = P_H \frac{\partial H}{\partial L_H} \cdot \quad (7)$$

But efficient production for the society would demand input allocation be so as to:

$$\text{Max } P_A A + P_H H$$

subject to:

$$L_A + L_H = \bar{L}$$

and wage rate at optimum would be ω^* so that:

$$P_H \frac{\partial H}{\partial L_H} = \omega^* = P_A \frac{\partial A}{\partial L_A} + P_H \frac{\partial A}{\partial H} \cdot \frac{\partial H}{\partial L_A} \quad (8)$$

Comparing (7) and (8), it can be seen that misallocation would occur in the competitive market because of the hidden effect of A on H.

c. The assumption about production functions and indifference curves having well defined and continuous curvatures is only necessary for the calculus technique to be used, which is a powerful technique when it is suitable, and the optimum conditions can be presented in a more simple and readily explanatory fashion. Otherwise it is not essential to the determination of the results. In a world of flat-faced, sharp-cornered production functions and indifference curves, linear programming would be a very effective analytical tool.

d. Increasing return to scale production can jeopardize the convexity property and leads to trouble. The main problem of increasing return to scale is that the optimum decision rule would generally require the activity to be carried out at loss when the total imputed factor incomes will exceed the total value of output.

Now, the outputs, so far discussed, belong to the class of privately consumed commodities. Samuelson (Ref. 9) generalized the problem further to include the class of collectively consumed commodities called public goods. If X_k is the amount of the public good available and x_k^i is the amount consumed by individual i, then

$$x_k^i = X_k$$

Samuelson then formulated the problem for the case of s individuals, n private goods, and m public goods as:

$$\text{Max } W(U^1, \ldots, U^s)$$

subject to:

$$F(X_1, \ldots, X_n, X_{n+1}, \ldots, X_{n+m}) = 0 .$$

where X_i, $i = 1,\cdots,n$ denotes the private goods

x_{n+j}, $j = 1,\cdots,m$ denotes the public goods.

By making the usual assumptions so as to make it possible for the Lagrangian technique to be used, the first order conditions would lead to:

$$\frac{U^i_j}{U^i_k} = \frac{F_j}{F_k} \quad (9) \qquad \begin{array}{l} i = 1,\ldots,s \\ j,k = 1,\ldots,n \end{array}$$

$$\frac{W_i U^i_j}{W_v U^v_j} = 1 \quad (10) \qquad \begin{array}{l} i,v = 1,\ldots,s \\ j = 1,\ldots,n \end{array}$$

$$\sum_{i=1}^{s} \frac{U^i_{n+j}}{U^i_r} = \frac{F_{n+j}}{F_r} \quad (11) \qquad \begin{array}{l} i = 1,\ldots,s \\ j = 1,\ldots,m \\ r = 1,\ldots,n \end{array}$$

All notations have the usual meanings.

(9) and (10) are the familiar results obtained previously for private goods only. (11) refers to the case of public goods which says that the sum of the marginal rate of substitution in exchange between the public good $(n+j)$ and the private good r, over the individuals, must be equal to the production substitution rate.

So as not to make the discussion too long, it suffices here to state that in an economy with money as the medium of exchange, the efficiency conditions (not counting the public goods case or, for that matter, externalities) will be achieved if the following price patterns are satisfied:

1. Prices of each class of variable inputs and outputs to be uniform over the person and production sectors

2. Prices of fixed supply and of intermediate goods to be uniform over the production sector

3. Personal consumption patterns to be so arranged that subjective substitution rates between goods equal their relative prices

4. Outputs and inputs to be increased in each industry until the price of the product equals its marginal cost. (Note: If all inputs supplied are inelastic then it is only necessary that prices be proportional to marginal costs in the same proportion.)

A more detailed discussion can be found in Millward (Ref.7).

From the above requirements, it is seen that if the economy consists of a strictly private enterprise system, which is purely competitive, where the firms maximize profits and individuals maximize utilities, then the efficient conditions would be automatically achieved. (Monopoly would fail to behave according to the efficient conditions unless all supplies are inelastic and the demand for the different products exhibits the same degree of elasticity since in

that case all prices would be of the same proportion to the corresponding marginal costs, hence efficient conditions are maintained.)

The above statement is true only if the following abnormal cases are excluded:

a. Several consumers have bliss point lying in the feasible set. In Figure 13, X_o is a Pareto optimal point; it is a competitive equilibrium point only if A is constrained to moves on the right of the budget line and B to the left. This is the case only if prices are negative. (X_A, X_B are the bliss points for A and B respectively.)

b. One consumer has a bliss point and indifference curves contain straight line segments. In Figure 14, the set of Pareto optimal points is the segment X_B B (X_B is the bliss point for B). However if the initial holding point is X_o and the budget line does not coincide with U_3^B, then X_o is the competitive equilibrium point (prices are positive) but is clearly not a Pareto optimal.

c. Some consumer is not able to trade because he holds no unit of any commodity that is desired by other consumers (i.e. what he owns, nobody wants). In Figure 15, B is satiated with respect to good 1 at the point x_{1B}^o, the initial joint holding is X_o, the set of Pareto optimum points is X_oB but X_o is not a competitive equilibrium point no matter what budget lines pass through X.

d. Commodities are indivisible. In Figure 16, commodities 1 and 2 are indivisible so that only the points shown

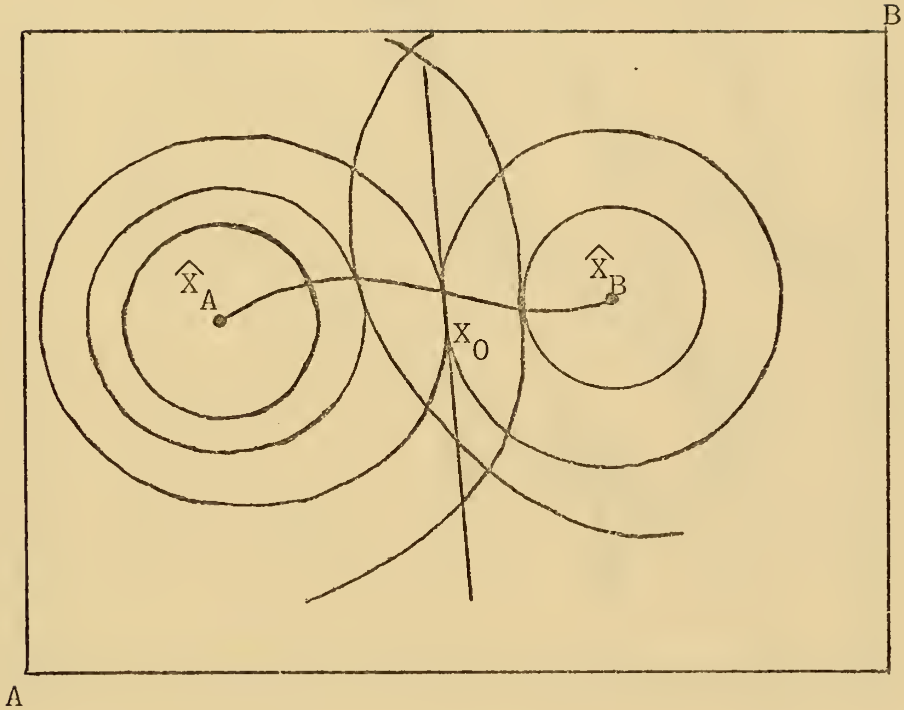

Figure 13

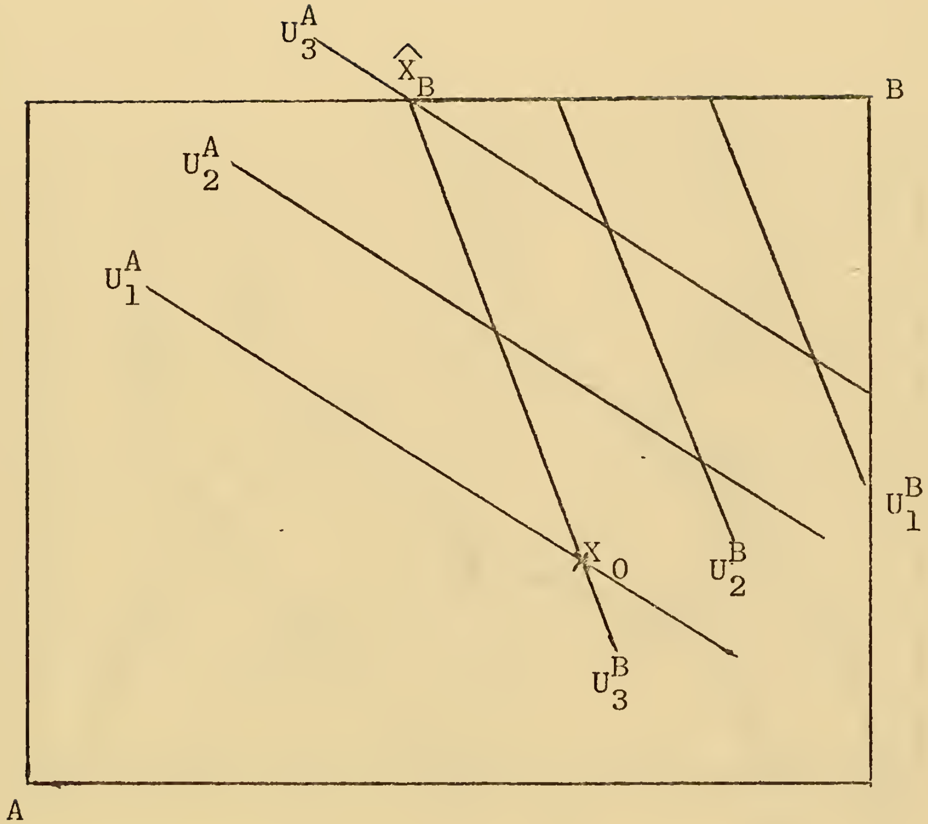

Figure 14

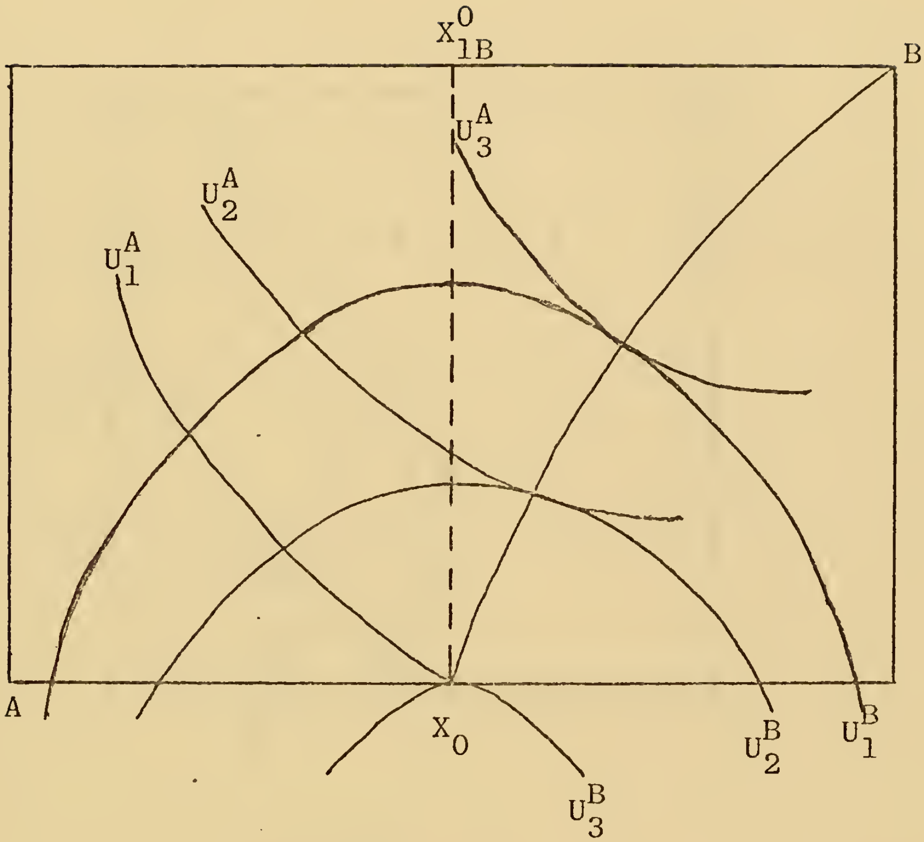

Figure 15

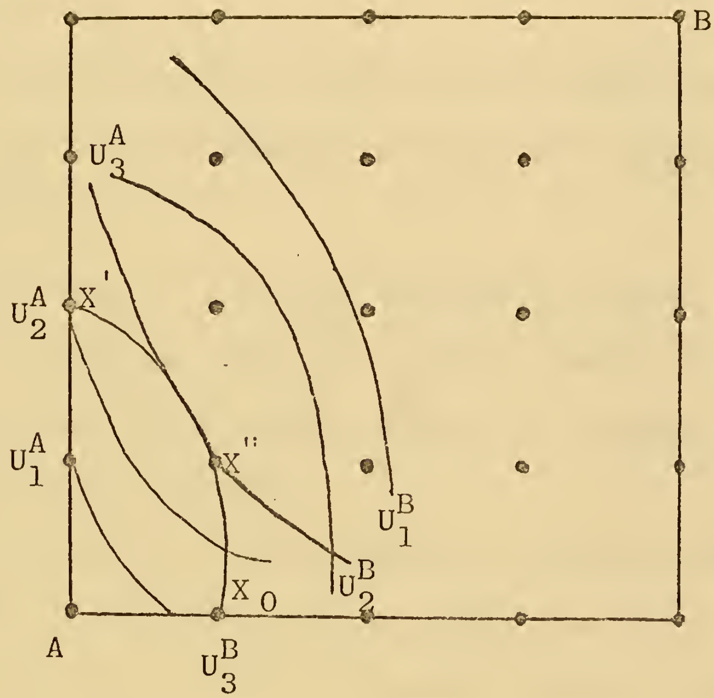

Figure 16

as "lattice-points" are feasible. Although preference curves are defined for all values of goods 1 and 2, A and B are only allowed to choose the lattice points in maximizing their utilities. Thus with the budget line shown where X_o is the initial joint holdings, the utility maximizing holding point for A and B which is consistent with market clearing and feasibility is X'. However X" is Pareto superior to X'.

e. Externality and public good, where either there is no way to price the product or the cost cannot be fully appropriated.

In the case of a social ownership system, then:

- As long as there is a large number of buyers and the market is allowed to clear, prices of outputs will be uniform over the person sectors.

- For inputs, large numbers of buyers and sellers on both sides of the market will ensure that all participators are price takers. With the market clearing, the prices on both sides are equated.

- As far as the person sector is concerned, utility maximization will guarantee that subjective substitution rates between goods be equal to their relative prices.

- For the production sector, it is required that the production of each commodity be pursued up to the level where price equals marginal cost.

In a mixed system where there are both private and public sectors, we would essentially need the combination of

the above conditions where it is relevant so that both sectors can satisfy the Pareto optimum conditions while in equilibrium.

Finally, it is worth summarizing a few main points about Pareto optimum conditions:

- That they are only necessary conditions and by merely operating in this condition in no way guarantees that the ultimate social welfare is maximized.

- That they are of all-or-nothing nature, i.e. if someone's behavior deviates from the Pareto condition then there is no longer any justification for others to obey this rule. Instead the problem of resource allocation would be approached in the manner of second best, i.e. to maximize the social welfare subject to the additional constraint due to the deviating behavior.

These points, coupled with the qualifications needed for arriving at Pareto optimum and the assumption that individual has perfect knowledge of what is best, really weaken the significance of trying to achieve Pareto optimum as compared with just any arbitrary behavior.

How good is it then?

This is best expressed by Mishan, and to quote him,

> "...Nevertheless, though an optimum per se cannot be vindicated as a norm to be pursued, some virtue may be detected in the 'lower level' optima of exchange and production.
>
> ...Irrespective of the distribution of welfare, a movement to or toward an exchange optimum is an unambiguous actual improvement in the welfare of the economy, i.e. some people will always be better-off and none worse-off if exchange between individuals of their initial product endowments is permitted.

...A production optimum is less reliable. A movement to or toward an optimum production point is an unambiguous potential improvement with certainty only if it entails an increased production of at least one good without reducing the production of another good.

...If, therefore, one disregards allocative criteria to the extent of trespassing upon these lower level optimum conditions, the welfare of the community is liable to be damaged."

IV. PUBLIC ENTERPRISE PRICING

We have discussed previously that, subject to a set of qualifications, Pareto efficient condition means that goods and services should be produced up to the levels where prices are equal to marginal costs no matter who is producing them, private or public enterprises.

The question is why the existence of public enterprises? The answer is public attitude toward government action, feasibility and efficiency.

In the normal situation, where the public has trust of their government, it is only natural that government activities in producing particular goods and services are favorably looked upon. After all the government is, to quote A. Lincoln, "...of the people, by the people, for the people ". It professes to follow the objective function of maximizing social welfare.

In underdeveloped or sometimes in developing countries, it is usually the case that government is the only one who can raise capital and provide the organization base to do the job of providing certain goods and services.

When externalities exist between different activities, private enterprises usually are not in as good a position as public enterprises - due to the limits on the scope of their activities - to capture the externality effect so as to come up with the true social cost of production necessary to

arrive at the right Pareto efficient condition. Private enterprises would also fail to behave according to the Pareto efficient condition where the market mechanism cannot lead to pure competition.

In the case of public goods, for example where consumption is non-excludable (e.g. defense, police), it is impossible to establish any appropriate pricing policy, since it is to the individual's advantage not to reveal the real value of the consumption. Also it may be doubtful that individual really knows how much is best for him. Thus the demand curve cannot be established. In this case, government is the only appropriate supplier, not only because it is in a better position to judge the need but also because its objective is not in profit but in social welfare.

Even if the exclusion principle could be made to work for consumption, the additional cost could be so large as to make the activity very ineffective (uneconomical), or that it is better to make the goods or services free, then again, government is in the position to be able to do it.

When a producer's product makes up a large portion of the market so as to allow it to manipulate the price, then a private monopolist would normally fail to follow Pareto efficient condition in his effort to maximize profit (except under the very special situation mentioned previously.)

For goods classified as public utilities (electricity, telephone, water, etc.) the economics and physical characteristics lead to efficient operation only under monopoly

(sometimes referred to as natural monopoly). For example, public utilities are characterized by large investment cost and very small operating cost giving rise to decreasing average unit cost. This efficiency due to size would make competition very unstable and eventually would lead to monopoly. Also by not having to worry about competition, the monopolist is more willing to develop bigger and more efficient plants, taking advantage of technological advancement and hence making fuller use of the contiunously decreasing average cost characteristics. Furthermore, monopoly will prevent duplication of facilities which is not only a waste economically but also causes physical obstruction (e.g. power lines, gas pipes, etc.). Another important feature of public utility that is favorable toward monopoly is that the commodities are, in general, non storeable. Goods and services must be produced when the order is made, which poses the problem that the production capacity must be large enough and must be efficiently used. Now individual demand normally exhibits strong fluctuations in different manners depending on what uses are made of the commodities. By gathering all various demands in one market, more even characteristics can be expected from the aggregate demand which can be better adapted to a given capacity, thus better efficiency is obtained.

In short, whenever accurate social pricing is not possible, government is the only desirable producer and whenever this pricing is possible, government may well be in

a better position to assess the cost and being subjected to much less financial constraint it has a better chance to adopt the desirable pricing policy while still being able to take advantage of any favorable characteristics related to the activity.

This is, of course, not to say that there is no place for private enterprises in producing goods and services. When the conditions allow the market mechanism to operate fairly successfully in the sense of pure competition, then it is a fairly costless and effective mechanism to lead activities to optimum condition no matter what types of enterprises are involved. Also, in aspects such as motivation and awareness, it is often the case that they exist in much stronger degree in the private sector than the public sector.

Next we would like to pose the question how should public enterprises behave?

This question has been the subject of many articles by economists. They fall in between the two "extreme" cases, one using the social welfare function of the Samuelson type (also called individualistic social welfare function) as the objective for the public sector, and the other making use of the compensation principle, with supply and demand functions being explicitly expressed in the objective function of the public sector.

For those using the social welfare function typified by an article by Boiteux (Ref. 10), attempts have been made to obtain pricing policy without the need of specifying the

exact form of the social welfare function except for some general properties such as:

- Smooth indifference curves so as to make calculus analysis possible

- Social welfare is increased if one individual's utility increases while others' remain unchanged (i.e. $\frac{\partial W}{\partial U^i} > 0 \quad \forall i$)

In other words, although the welfare function is used, the interpretation is limited to the Pareto sense only.

In the case of Boiteux, he formulated the problem considering an economy of n commodities (input being negative, output positive), a private sector with v producers, a public sector with w producers and the consumers of m individuals.

The consumers' behavior is formulated as:

$$\text{Max } U^k (Q^k)$$

subject to: (12) $k = 1,\ldots,m$

$$P^t Q^k = r^k$$

where $Q^k = (q_1^k, \cdots, q_n^k)^t$, the bundle of commodities supplied (if negative) or consumed (if positive) by individual k,

r^k = the lump sum tax (if negative) or subsidy (if positive) on individual k,

$P^t = (p_1, \cdots p_n)$, the price vector.

The private enterprises are operating under pure competition. Each firm's behavior is formulated as:

$$\text{Max } P^t X^h$$

subject to: (13) $h = 1, \cdots, v$

$$f^h(X^h) = 0$$

where $X^h = (x_1^h, \cdots x_n^h)$, the bundle of commodities used (if negative) or produced (if positive) by the firm h.

Each public enterprise has production function

$$g^l(Y^l) = 0 \qquad l = 1, \ldots, w$$

and a budget constraint

$$P^t Y^l = b^l$$

where $Y^l = (y_1^l, \ldots, y_n^l)$, the bundle of commodities used (if negative) or produced (if positive) by the enterprise l,

b^l = the constrained profit (if positive) or deficit (if negative).

The market is allowed to clear, i.e.

$$\sum_{k=1}^{m} q_i^k = \sum_{h=1}^{v} x_i^h \quad \sum_{l=1}^{w} y_i^l \qquad i = 1, \ldots, n$$

Boiteux expressed the social welfare function in the form of

$$W = \sum_{k=1}^{m} \lambda_k U^k$$

and formulated the public sector behavior as:

$$\operatorname{Max} \sum_{k=1}^{m} \lambda_k U^k$$

subject to:

Consumers behavior as in 12

Private enterprises behavior as in 13

Market clearing constraint

Public enterprises production and budget constraints

Now, consumers behavior would result in $U^k(P,r^k)$ and $q_i^k(P,r^k)$. Private enterprises behavior would result in $x_i^h(P)$. Hence the above problem could be put in the form:

$$\operatorname{Max} \sum_{k=1}^{m} \lambda_k U^k(P,r^k)$$

subject to:

$$\sum_{k=1}^{m} q_i^k(P,r^k) - \sum_{h=1}^{v} x_i^h(P) - \sum_{l=1}^{w} y_i^l = 0.$$

$$g^l(y^l) = 0.$$

$$b^l - P^t y^l = 0.$$

The Lagrange function would be:

$$L(P,r^k,y^l) = \sum_{k=1}^{m} \lambda_k U^k(P,r^k) + \sum_{i=1}^{n} \mathcal{K}_i \left(\sum q_i^k \quad \sum x_i^h \quad \sum y_i^l\right) + \sum_{l=1}^{w} \varphi_l\, g^l(y^l) + \sum_{l=1}^{w} \beta_l (b^l - P^t y^l)$$

By manipulating first order conditions, Boiteux got to the result:

$$\sum_{i=1}^{n-1} z_i(ij) = \sum_{l=1}^{w} \beta_l y_j^l \qquad (14) \qquad j = 1,\ldots,n-1$$

where $$(ij) = \sum_{k=1}^{m}\left(\frac{\partial q_i^k}{\partial p_j} + q_j^k \frac{\partial q_i^k}{\partial r^k}\right) - \sum_{h=1}^{v} \frac{\partial x_i^h}{\partial p_j}$$

$$= \sum_{k=1}^{m} (ij)^k - \sum_{h=1}^{v} (ij)^h$$

(ij) is called the global coefficient of substitution of goods i and j for the household and industrial consumers (suppliers) taken together.

$$z_i = \mu_i - \mu_n (p_i / p_n)$$

$$= (\beta_l - \mu_n / p_n)(p_i - g_i^l p_n / g_n^l)$$

Note: $z_n = 0$

Since the μ's coefficients appear in homogeneous form (first degree), it is permissible to let $\mu_n = -p_n$, then:

$$z_i = (1 + \beta_l)(p_i - g_i^l p_n / g_n^l)$$

$$= (1 + \beta_l) t_i^l \quad , \quad t_i^l = p_i - g_i^l p_n / g_n^l$$

Now:

1. If no budget constraint is required, $\beta_l = 0 \quad \forall l$

   Then since the matrix $\{(ij)\}$ is non-singular:

$$z_i = 0 \qquad i = 1,\ldots,n-1$$

$$\text{or} \quad p_i = g_i^l \, p_n \,/\, g_n^l$$

i.e. price is equal to marginal cost.

2. If there is only one global budget constraint for the public sector

   i.e. $b - p^t Y = 0$ where $Y = \sum_{l=1}^{w} Y^l$

and let β be the associated Lagrange multiplier, then (14) becomes

$$\sum_{i=1}^{n-1} z_i (ij) = \beta y_j \quad (15) \qquad j = 1, \ldots, n-1$$

which leads to:

$$t_i^l = \frac{z_i}{1+\beta} = t_i \qquad \ell = 1, \ldots, w \quad i = 1, \ldots, n-1$$

hence $g_i^l p_n / g_n^l = g_i^s p_n / g_n^s = \pi_i$

but $\pi_n = p_n$

$$\therefore \quad \frac{g_i^l}{\pi_i} = \frac{g_n^l}{\pi_n} = \frac{g_i^s}{\pi_i} = \frac{g_n^s}{\pi_n}$$

Thus all public enterprises must act as though they are maximizing their profits with respect to the fictitious price system $\pi = (\pi_1, \ldots, \pi_n)$ common to all of them (except for commodity n since $p_n = \pi_n$).

Note that $\pi_i = g_i^1 p_n / g_n^1$ is the marginal cost of producing good i, but the selling price is p_i where

$$p_i - \pi_i = t_i$$

Thus the public enterprises are operating at the output levels where the selling prices deviate from their marginal costs. If these deviations are not too large, in the sense as to allow differential expressions to be used for changes (Ramsey (Ref. 11) also stressed this point), then by viewing t_i as p_i it could be shown that

$$\sum_{i=1}^{n-1} (ij) t_i = \sum_{i=1}^{n-1} (ij) \delta p_i = \delta y_j$$

Compared with (15) we have:

$$\delta y_j = \frac{\beta}{1+\beta} y_j = \eta y_j \qquad j = 1,\ldots,n-1$$

Thus when the public sector is subjected to a single global budget constraint then except for the reference product whose price is fixed ($\pi_n = p_n$), all others must be sold at prices deviated from marginal cost (or output must be at the level such that its marginal cost deviates from the market price) in such amounts as to cause the same relative change - if the change is "small" - in outputs from the optimum levels when no budget constraint is imposed at all, provided a change in prices is accompanied by a compensated variation in r^k.

3. If each public enterprise has its own budget constraint then since

$$z_i = (1+\beta^l)\, t_i^l$$

we have:

$$\frac{t_i^l}{t_j^l} = \frac{t_i^s}{t_j^s} = \frac{z_i}{z_j} \qquad \begin{matrix} l, s = 1,\ldots,w \\ i, j = 1,\ldots,n-1 \end{matrix}$$

Thus except for the reference good, the relative deviations between goods are the same for all public enterprises.

Define:

$$\sum_{i=1}^{n-1} z_i^l\,(ij) = y_j^l \qquad (16) \qquad j = 1,\ldots,n-1$$

Then from (14) we have

$$t_i^l = \frac{\sum_{s=1}^{w} \beta_s z_i^s}{1 + \beta_l} \quad (17) \qquad \begin{array}{l} l = 1, \ldots, w \\ i = 1, \ldots, n-1 \end{array}$$

(16) is similar to (15) and thus for given good i (except for the reference good) a public enterprise 1, operating under its own budget constraint, would produce at a level such that the deviation of price from marginal cost t_i^1 would be as in (17) where z_i^1 would be proportional to small changes in prices causing the same relative change in the commodities produced (consumed) by enterprise 1.

Note the important assumptions used by Boiteux to arrive at the above results:

- Private sector is operating under pure competition
- Lump sum tax is possible (r^k)
- A change in price is accompanied by a compensated change in r^k.

In similar vein, Mohring (Ref. 12) however concentrated his analysis on the peak load problem associated with most public utility. He restricted the peak load problem to only two periods, the peak period demand which he called X_1, the off-peak demand X_2 and a third good X_3 so as to allow the problem to be viewed in the light of the total economy without having to make it too complicated.

Mohring treated cost functions in terms of the cycle-period outputs. Thus let α_i (i=1,2) denote the fraction of time the demand is of type i, since the actual output is X_i

if the whole period had been used to produce good i only then x_i/α_i would have been produced at a cost $C(X_i/\alpha_i, K)$ where K is the annual cost of the public utility's capital plant, then the operating cost of providing X_i is $\alpha_i C(X_i/\alpha_i, K)$.

X_3 was treated by Mohring in a very particular way. It is a numeraire good and also its production is such that a unit of resource services can be converted to a unit of X_3. Thus X_3 serves not only as a reference good but also as the type of "final" outputs as seen by Lerner (e.g. leisure).

As for the consumers - in Mohring's model, it is convenient to regard the sole resource service as labour for ease of interpretation - each can choose to work a certain amount to pay for the consumption of goods 1 and 2 and the lump sum tax (subsidy) and the rest of their labour resource to be converted to leisure, namely good 3.

The consumer behavior is thus:

$$\text{Max } U^i(x_1^i, x_2^i, x_3^i)$$

subject to:

$$r^i - h^i = p_1 x_1^i + p_2 x_2^i + p_3 x_3^i \qquad i = 1, \ldots, n$$

where r^i is individual labor resources, h^i is his head tax (subsidy) and there are n individuals in the economy.

The public utility behavior is to:

$$\text{Max } W(U^1, \ldots, U^n)$$

subject to:

$$R = \alpha_1 C_1 + \alpha_2 C_2 + X_3 + K$$

where

$$R = \sum_{i=1}^{n} r^i$$

$$C_i = C(X_i / \alpha_i , K) \qquad i = 1, 2$$

$$X_j = \sum_{i=1}^{n} x_j^i \qquad j = 1, 2, 3$$

and W is the individualistic social welfare function.

Mohring's results are summarized as follows:

1. If no budget constraint is imposed then the conclusion is again that the utility should operate at the point where price and marginal cost are equal.

2. If a budget constraint is adopted and if the products in the two periods are substitute, then price should be greater than marginal cost in both periods (shifting effect of demand). If they are complements then it may be the case that price in the off-peak period may be lower than its marginal cost.

3. For the case when it is not possible to price differently the commodities in the periods, a single price constraint is introduced. The result is a single price which is a combination of the marginal costs in both periods.

In all three cases mentioned above, the optimum capacity level would lead to the saving possible in the variable cost due to an increase in capacity being exactly offset by the additional cost of increasing the capacity.

4. For the case of single toll constraint, Mohring's example is the road problem where the variable cost is born by the consumer himself; the only price that the public utility can impose on the consumers is gasoline tax. Thus the commodity's price is of the form

$$P_j = T + \alpha_j C(X_j/\alpha_j, K)/X_j$$

In this case optimum capacity may result in it being "inefficiently" small or large (by an inefficient size is meant that the saving in the variable cost can no longer be offset by the extra cost required to increase the capacity).

Thus, apart from studying the problem of public enterprises' behavior under various other types of constraint, Mohring's approach is essentially the same as Boiteux's:

- The same general welfare function
- Outputs being X_1, X_2, X_3
- Input being r^k
- Lump sum tax (subsidy) being h^k
- Individuals are price takers and utility maximizers

The difference is that in Boiteux's case prices are set by a purely competitive market mechanism and government behavior is to influence the output levels and the associated marginal costs while manipulating the lump sum tax to satisfy any income redistribution required by the social welfare function, whereas in the Mohring model government behavior is to influence output levels and the associated prices while manipulating the lump sum tax to obtain the

desired income distribution. Mohring also studied the question of optimum production capacity which is an important aspect in public utilities analysis.

The above approach of using the Samuelson type of social welfare function makes it necessary that the analysis be done with models covering the whole economy with all producers and consumers being taken into account (although they may be assumed to consist of only a few). It is conceptually the most appropriate and general but this is at the price of heavy burden of mathematical derivation and manipulation. Many neat and easily interpretable results are possible only due to the use of lump sum taxes giving rise to many additional first order conditions aiding the mathematical manipulations and thus results in simple formulae are obtained.

At the other extreme, some economists wanted to give the public enterprises a much more explicit type of objective function. To do this, the compensation principle was used while assuming that the effect of the public enterprise activity under consideration is totally contained within its sphere of influence, i.e. the behavior in the economy external to the public enterprise activity is unaffected and vice versa. Thus the gain or loss associated with the activity can be established without the need to enlarge the problem to cover the whole economy.

The compensation principle, in this context, says that an activity is favorable if there is a net gain associated with it. Thus the objective of the public enterprise is to

maximize this net gain. Dupuit (Ref. 13), Marshall suggested measuring a consumer's gain as the difference between the price he is willing to pay and the actual price paid to acquire a unit of the product and a producer's gain as the difference between the revenue he can collect and the cost involved in producing that level of output. Then, assuming also perfect divisibility, it can be seen that the net gain excluding the fixed cost is measured as the area between the demand and the marginal cost curves in the range of the output being produced and that the area reaches its maximum value when the output is at the level where the demand and marginal cost curves intersect, i.e. where price is equal to marginal cost. Figure 17 illustrates this point.

This view involves the assumption that the demand for the products produced by the public enterprises are independent of those belonging to the economy external to them, that there is no income effect. Little (Ref. 14) and many others showed that for the measure of willingness-to-pay to be consistent (i.e. the same amount is obtained, irrespective of how the consumer chooses to respond to changes in quantity or price) then the marginal utility of money has to be constant. Samuelson (Ref. 15) has shown that there is no hope for the conditions - constant marginal utility of money, independent utility contribution - to be satisfied completely. The surplus concept is to him "worse than useless."

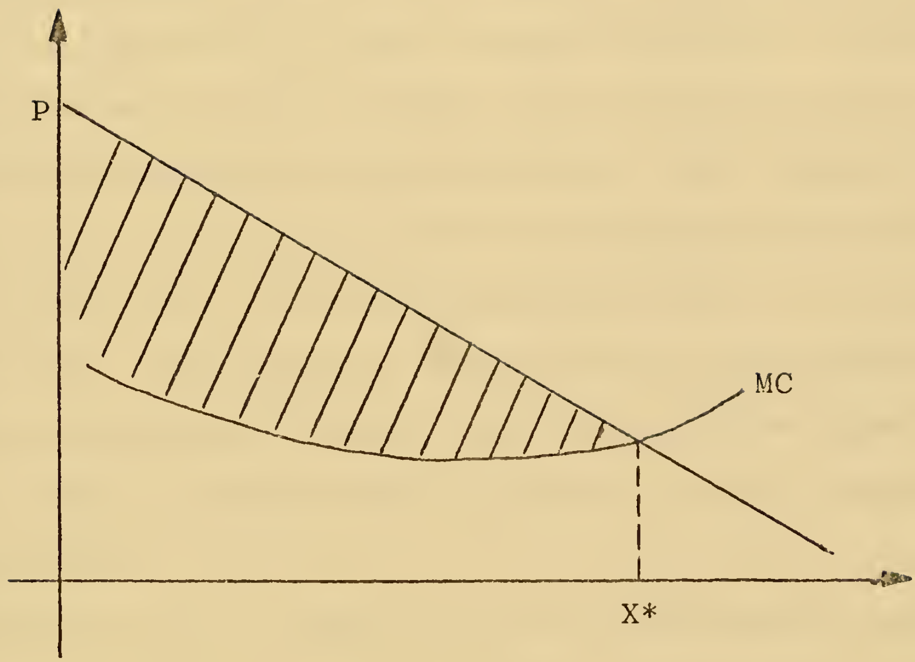

Figure 17

Proponents for the surplus approach argued that it can be useful as good approximations. If the amount of income involved in the consumption of the products under consideration is only a small part of the total budget, then it will not noticeably change the pattern of consumption on other goods and hence the marginal utility of money can be considered as constant.

Following Dupuit, one can view the net gain (or net benefit) as a measure of gain in social welfare. In this case more assumptions are clearly involved such as that utility is measurable in terms of money, that everyone is treated in the same way and that the consumption of the products contributes its own independent amount of utility. Johansen (Ref. 16) used such an approach in his discussion on public activities. In this case the objective function is again readily interpretable as social welfare, although of a special kind.

Alternatively, as suggested by the compensation principle, the net gain is the net earning above variable cost that the public suppliers can obtain if perfect discriminating pricings are possible. This amount can then be used to recover the fixed cost and the rest will be profit. In this circumstance, the consumers will be indifferent but the public suppliers will gain profits, i.e. some one could be better off and none worse off. There is then no need to assume measurable utility (i.e. utility is cardinal in nature) or that everyone be treated in the same way. Thus the goal for the public

enterprises would be to maximize the potential profit which, when realizable, will leave every consumer indifferent.

It is important to emphasize at this point that, just as with private enterprises in a competitive market whose goal is to maximize profit, price being equal to marginal cost is only a necessary condition. A private enterprise must at least recover its total cost. In the case of public enterprises, the net gain as defined must be bigger than the related fixed cost to make the operation desirable. This is relevant to such decisions as whether the public enterprise should be established to function at all (i.e. decision to start the activity) or whether an expansion in the activity should be made which requires additional capital investment.

Now, originally the net benefit concept was developed for each single product in the public sector and hence required the assumption that the demand for each product be independent, i.e. there is no cross elasticity. Hotelling (Ref. 17) generalized the concept to cover the case where the products exhibit cross elasticity , thus need to be considered together. The generalized consumer surplus function is then of the form:

$$\left[\int_0^{q^*} \sum_{i=1}^{n} p_i(q)\, dq_i \right] - \sum_{i=1}^{n} p_i^* q_i^*$$

where p_i is the price of good i when the consumption is bundle $q = (q_1,\ldots,q_n)$, and p_i^* is its price when the consumption bundle is $q^* = (q_1^*,\ldots,q_n^*)$.

The integrability condition for the preceeding expression requires that

$$\frac{\partial P_i}{\partial q_j} = \frac{\partial P_j}{\partial q_i}$$

the economic meaning of which is that there be no income effect between those goods.

Pressman (Ref. 18) provided quite a typical and general analysis of pricing policy for the public sector using the surplus concept, although his article was mainly concerned with the problem of peak load pricing.

Pressman considered a public utility offering its good for the peak and off-peak demands in amounts q_1 and q_2 respectively, with cost functions

$$D_1\ (q_1,K) = D\ (q_1,K)$$
$$D_2\ (q_2,K) = D\ (q_2,K)$$

where K represents the utility capacity, and the capacity cost is $\bar{q}$ (K). The total cost function is then:

$$C(q_1,q_2,K) = D_1(q_1,K) + D_2(q_2,K) + \bar{q}(K).$$

Pressman analyzed the problem with regard to two possible constraints: profit and capacity. Thus the mathematical formulation is:

$$\text{Max} \left[\int_{(0,0)}^{(q_1^*,q_2^*)} \sum_{i=1}^{2} P_i\, dq_i \right] - C\,(q_1^*, q_2^*, K)$$

subject to: $$\sum_{i=1}^{2} P_i q_i^* - C(q_1^*, q_2^*, K) \leq M$$

$$q_i^* \leq K \qquad i = 1,2$$

$$q_i^* \geq 0 \qquad i = 1,2$$

$$K \geq 0$$

By investigating the Kuhn-Tucker condition for maximum solution, he was able to derive the rollowing results:

In the general case where the demands are independent then:

- The capacity is optimum when

$$-\frac{\partial C}{\partial K} = \frac{\partial D_1}{\partial K} + \frac{\partial D_2}{\partial K} + \frac{\partial \bar{q}}{\partial K} = 0$$

which is the same as Mohring's result

- If there is no profit constraint then we obtain the familiar result:

$$P_i(q_1, q_2) = \frac{\partial D_i}{\partial q_i} \qquad i = 1,2$$

- If the profit constraint is imposed then

$$P_i(q_1, q_2) = \frac{\partial D_i}{\partial q_i} + \frac{\lambda}{1+\lambda}\left(\frac{1}{\epsilon_i} + \frac{1}{\epsilon_{ij}}\right) P_i(q_1, q_2) \quad j,i = 1,2 \quad i \neq j$$

where
$$\epsilon_i = -\frac{\partial q_i}{\partial p_i} \cdot \frac{P_i}{q_i} \quad , \quad \epsilon_{ij} = -\frac{\partial q_i}{\partial P_j} \cdot \frac{P_j}{q_i}$$

and λ is the Lagrange multiplier associated with the profit constraint.

If, in addition, the goods are independent then

$$p_i = \frac{\partial D_i}{\partial q_i} + \frac{\lambda}{1+\lambda} \cdot \frac{1}{\epsilon_i} \cdot p_i$$

- If the capacity constraints are imposed on both periods, then:

$$p_i = \frac{\partial D_i}{\partial q_i} + \frac{\lambda}{1+\lambda}\left(\frac{1}{\epsilon_i} + \frac{1}{\epsilon_{ij}}\right) p_i + \frac{\gamma_i}{1+\lambda} \qquad (18) \qquad i = 1,2$$

where γ_i is the multiplier associated with capacity constraint for period i.

In this case optimum capacity must be such that

$$\frac{\partial C}{\partial K} = \sum_{i=1}^{2} \gamma_i \qquad (19)$$

Now, if there is no longer a profit constraint, i.e. $\lambda = 0$, then

$$p_i = \frac{\partial D_i}{\partial q_i} + \gamma_i \qquad (20) \qquad i = 1,2$$

Furthermore if, for definiteness, only period 2 is under capacity constraint then

$$\frac{\partial C}{\partial K} = \gamma_2$$

$$p_1 = \frac{\partial D_1}{\partial q_1} \qquad (21)$$

$$p_2 = \frac{\partial D_2}{\partial q_2} + \gamma_2$$

Pressman also worked with a linear cost model where

$$D_1 = bq_1 \quad , \quad D_2 = bq_2 \quad , \quad \bar{q}(K) = \beta K$$

The result of the analysis for this case can be derived from the above general case.

By using the surplus concept for the public enterprise's objective function, Pressman has been able to derive results which are of the form given us by Mohring. They are typically represented by (18) and (19).

From (19) it is interesting to note that when no capacity constraint exists (i.e. $\frac{\partial C}{\partial K} = 0$) then the capacity should be extended to the optimum value when the saving in operating marginal cost ($\sum_{i=1}^{2} \frac{\partial D_i}{\partial K}$) is just offset by the extra cost needed to acquire more capacity. This situation is relevant to the decision of expanding the capacity by using new plant (thus obtaining lower operating cost). This point was discussed by Millward (Ref. 19), but in the context of the indivisibility of extra capacity and the time dimension to cost was also accounted for and thus the result was expressed in terms of equivalent discounted value.

When the extra capacity does not affect operating cost (i.e. $\frac{\partial D_i}{\partial K} = 0 . \forall i$), for example the linear model shown above, and that it requires some cost to have extra capacity then 19 says that either γ_1 or γ_2 (whichever is related to peak period) or both cannot be zero. Thus, at least the peak demand must be subjected to capacity limit for efficient plant size.

Consider $C(q) = D(q) + \bar{q}(K)$, where the capacity limit is represented by K. When $q = K$ then

$$C(K) = D(K) + \bar{q}(K)$$

$$\frac{dC}{dK} = \frac{dD}{dK} + \frac{d\bar{q}}{dK} \qquad (22)$$

(22) represents the long term marginal cost, Boiteux called it, the expansion cost.

By examining (18), (19), (22), it can be seen that when there is no profit constraint ($\lambda = 0$) and only peak demand is under capacity limit, the optimum limit of capacity should be so, that the peak demand price is equal to the expansion cost. When both period demands are subjected to the capacity limit then their prices exceed the operating marginal costs by their share $_i$ of the capacity cost which Boiteux called the development cost. These results were also obtained by Boiteux (Ref. 20) and Steiner (Ref. 21).

(18) says that, when the profit constraint exists, the price deviation contains another component which should be proportional to the price and $(\frac{1}{\epsilon_i} + \frac{1}{\epsilon_{ij}})$. When there is no cross elasticity (i.e. no $\frac{1}{\epsilon_{ij}}$ term) then the more inelastic the demand the larger is the deviating component imposed by profit constraint. When the demand is completely inelastic then the fixed cost should be covered by this demand of the product alone, unless other consideration is taken into account, e.g. if the customers are poor people then it would not be desirable to follow this policy.

Note again that the relative smoothness in achieving the above results is at the expense of making some strong assumptions concerning surplus analysis. When the surplus is interpreted as a measure of net gain in social welfare then it implies some further assumptions to be made such as that utility is measurable in terms of money; that society weights individual utility equally, on the other hand it removes any ambiguity that is there in the objective function. However by interpreting the surplus as the "potential profit" over the variable cost, then the "improvement" due to public sector operation is only in the sense of the compensation principle which we have seen to contain some possible ambiguity even in its "loose" sense on improvement (i.e. potential improvement).

Now in between the two approaches just shown are those represented in articles by Baumol and Bradford (Ref. 22) and Ramsey (Ref. 23) where a particular form of social welfare function is used which only takes into account the aggregate consumption.

The results presented by Baumol and Bradford's analysis can be seen in their simple general equilibrium model. Here they proposed only one input resource, labor, and n outputs with cost function $F(x_1,\dots,x_n)$ to measure the required input resource, the social welfare function $Z(p_1,\dots p_n)$ where p_i is the unit price of product i. The production of $(x_1,\dots x_n)$ is to bring back a profit M. Thus the problem is formulated as:

$$\text{Max } Z(p_1, \ldots, p_n)$$

subject to:

$$\sum_{i=1}^{n} p_i x_i - F(x_1, \ldots, x_n) = M$$

The first order conditions lead to the following relation using

$$\frac{\partial Z}{\partial p_j} = -x_j$$

$$\sum_{i=1}^{n} \left(p_i + \lambda \frac{\partial F}{\partial p_j} \right) \frac{\partial x_i}{\partial p_j} = (1+\lambda) \left(x_j + \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j} \right) \qquad (23)$$

where λ is the Lagrange multiplier associated with the profit constraint.

Note that by expressing the welfare function this way and by assuming $\frac{\partial Z}{\partial p_j} = -x_j$, Baumol and Bradford essentially assumed that the marginal social welfare of individual income is the same for everyone, irrespective of size. Thus, if marginal individual utility of income is the same, then everyone's utility is weighted equally in the social welfare computation.

Let $$L = \sum_{i=1}^{n} p_i x_i$$

then $$\frac{\partial L}{\partial p_j} = x_j + \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j}$$

(23) can then be written as:

$$\sum_{i=1}^{n} \left(p_i + \lambda \frac{\partial F}{\partial p_j} \right) \frac{\partial x_i}{\partial p_j} = (1+\lambda) \frac{\partial L}{\partial p_j} \qquad (24)$$

$$-\frac{\partial L}{\partial p_j} = \sum_{i=1}^{n} \frac{\partial L}{\partial x_i} \cdot \frac{\partial x_i}{\partial p_j}$$

$$\sum_{i=1}^{n}\left(p_i + \lambda \frac{\partial F}{\partial x_i}\right) \frac{\partial x_i}{\partial p_j} = \sum_{i=1}^{n} (1+\lambda) \frac{\partial L}{\partial x_i} \cdot \frac{\partial x_i}{\partial p_j}$$

If no cross elasticity exists then

$$p_i + \lambda \frac{\partial F}{\partial x_i} = (1+\lambda) \frac{\partial L}{\partial x_i} \qquad i = 1, \ldots, n$$

which can be reduced to

$$p_i = \frac{\partial F}{\partial x_i} + \frac{1+\lambda}{\lambda} \cdot \frac{1}{\epsilon_i} \cdot p_i \qquad (25)$$

where

$$\epsilon_i = -\frac{\partial x_i}{\partial p_i} \cdot \frac{p_i}{x_i}$$

This is the same as Pressman's result.

When the deviation $p_i - \frac{\partial F}{\partial x_i}$ is small, then (25) leads to:

$$\delta x_i = k x_i \qquad (26)$$

which is the same as Boiteux's result. Baumol and Bradford claimed that (26) holds even if cross elasticity exists and cited Boiteux's proof. But Boiteux had used the compensated lump sum tax for changes in price to deduce the result in a general context, thus their claim that (26) holds generally is only true in this respect.

If L is fixed, i.e. the effect of tax results in resource reallocation within the taxed sector only then:

$$\frac{\partial L}{\partial x_i} = 0$$

and

$$p_i + \lambda \frac{\partial F}{\partial x_i} = 0 \qquad (27)$$

i.e. it is sufficient for prices to be proportional to marginal costs in the same proportion.

If the products concerned are of the Lerner types (i.e. "final" outputs in the sense that leisure is also a product) then L represents the whole economy which, in the short run, is indeed quite inelastic, then (27) is in fact Lerner's statement that in the case all "final" goods can be taxed, it is only necessary to make price in the same proportion to its marginal cost.

Baumol and Bradford took the view that since it is not feasible to tax every "final" goods in the Lerner sense, then only final goods that are in market transactions should be considered. In this case it is true that prices being in the same proportion to marginal costs satisfy optimum condition only if resource reallocation occurs only within the taxed sector.

Ramsey approached the problem in a different manner. He used the net utility function $U(x_1,...,x_n)$ which can be viewed as a social welfare function taking into account only of aggregate consumption (similar to Baumol, Bradford).

When there is no profit constraint, optimum outputs should be at a level such that

$$\frac{\partial U}{\partial x_i} = 0 \qquad i = 1, \ldots, n$$

Let the optimum quantities by $(\bar{x}_1, \ldots, \bar{x}_n)$. The profit constraint would then be introduced through the required amount of revenue to be collected through tax.

$$R = \sum_{i=1}^{n} \lambda_i x_i$$

The tax optimization problem would be:

$$\text{Max } U(x_1, \ldots, x_n)$$

subject to:

$$R = \sum_{i=1}^{n} \lambda_i x_i$$

Ramsey came up with the condition for tax optimization, assuming constant marginal utility of income so that

$$\frac{\partial U}{\partial x_r} = \lambda_r$$

and thus the optimum condition:

$$\frac{\lambda_1}{\sum_{s=1}^{n} \frac{\partial \lambda_1}{\partial x_s} x_s} = \frac{\lambda_2}{\sum_{s=1}^{n} \frac{\partial \lambda_2}{\partial x_s} x_s} = \cdots = \frac{\lambda_n}{\sum_{s=1}^{n} \frac{\partial \lambda_n}{\partial x_s} x_s} \qquad (28)$$

for λ_r "small" enough so that

$$\lambda_r = \sum_{s=1}^{n} \frac{\partial \lambda_r}{\partial x_s} \delta x_s \qquad (29)$$

Then (28) implies

$$\frac{\delta x_1}{\bar{x}_1} = \frac{\delta x_2}{\bar{x}_2} = \cdots = \frac{\delta x_n}{\bar{x}_n}$$

which is similar to Baumol, Bradford and Boiteux conclusions.

For the above results to remain true for large change in consumptions due to tax, it is sufficient that (29) remains true for large variation of x_r,

$$i.e. \qquad \lambda_r = \sum_{s=1}^{n} \frac{\partial \lambda_r}{\partial x_s} X_s \quad , \quad X_s = x_s - \bar{x}_s$$

which implies that λ_r is linear in x's.

Ramsey thus proposed a quadratic utility function in x's.

$$U = \text{Constant} + \sum_{r=1}^{n} \alpha_r x_r + \sum_{r=1}^{n} \sum_{s=1}^{n} \beta_{rs} x_r x_s$$

where $\sum_{r=1}^{n} \sum_{s=1}^{n} \beta_{rs} x_r x_s$ is negative definite

The optimum point when no revenue is collected would have to satisfy

$$\frac{\partial U}{\partial x_r} = \alpha_r + 2 \sum_{s=1}^{n} \beta_{rs} \bar{x}_s = 0$$

When tax λ_r is imposed then:

$$\lambda_r = \alpha_r + 2 \sum_{s=1}^{n} \beta_{rs} x_s$$

$$= 2 \sum_{s=1}^{n} \beta_{rs} (x_s - \bar{x}_s) = 2 \sum_{s=1}^{n} \beta_{rs} X_s$$

which is linear in x's.

In this case the utility function is represented by hyper-ellipsoids indifferent surfaces with the center at ($\bar{x}_s$, s=1,...,n) and the revenue levels by other hyper-ellipsoids with the center at ($\frac{1}{2}\bar{x}_s$, s=1,...,n).

Ramsey also considered the case where all commodities are independent and have their own supply and demand curves, thus

$$\lambda_r = p_r(x_r) - q_r(x_r)$$

Then for small changes in demands, the optimum ad-valorem tax (where $\lambda_r = \mu_r q_r$) should be such that

$$\frac{\mu_1}{\frac{1}{\epsilon_1} + \frac{1}{\rho_1}} = \frac{\mu_2}{\frac{1}{\epsilon_2} + \frac{1}{\rho_2}} = \cdots = \frac{\mu_n}{\frac{1}{\epsilon_n} + \frac{1}{\rho_n}} \qquad (30)$$

where ϵ_r and ρ_r are elasticities of supply and demand respectively, defined so that they are both positive.

From (30), if any commodity is completely inelastic, either in demand or supply or both, then the whole revenue should be collected on it. A result similar to other author's findings mentioned before.

When all commodities have independent demands but are complete substitutes in supply,

i.e. $$p_r = p_r(x_r)$$

$$q_r = q_r\left(\sum_{s=1}^{n} x_s\right)$$

then (30) becomes

$$\frac{\mathcal{M}_1}{\frac{1}{\epsilon} + \frac{1}{e_1}} = \frac{\mathcal{M}_2}{\frac{1}{\epsilon} + \frac{1}{e_2}} = \cdots = \frac{\mathcal{M}_n}{\frac{1}{\epsilon} + \frac{1}{e_n}}$$

when the supply is inelastic (i.e. $\epsilon \to 0$) then $\frac{1}{e_i} \ll \frac{1}{\epsilon}$ for all i and the ad valorem tax is the same for every good, as might be expected.

Different approaches to the problem of resource allocation in the public enterprise activities have been presented. We have seen that while the use of the Samuelson type of social welfare function as the objective function for the public sector is the most satisfying approach conceptually, it makes the analysis much more involved and, unless lump sum tax is assumed to take care of the income distribution problem, the results cannot be reduced to a readily interpretable form. To a lesser extent, the type of social welfare function that depends upon the aggregate consumption only helps to reduce the large number of variables involved. Furthermore, since this type of social welfare does not take into account individual consumption per se, it implies that marginal social welfare with respect to individual income is the same for all of them, irrespective of size. The analysis is then much simpler and the results are more readily interpretable. For both approaches, the framework is smooth

continuity and perfect divisibility. If an indivisibility is involved then the analytical tool will have to be mathematical programming, in which case the objective function would have to be completely specified. This is a complex task and so far it does not appear feasible to do so.

In contrast, the surplus concept allows the objective function to be expressed in terms of supply and demand functions which are intimately related to the activity under consideration. To make the most of this type of objective function one will normally restrict the analysis to the enterprises concerned. This is tantamount to the assumption that there be no externality between the enterprises and the economy external to them, otherwise the solution would only be the case of sub-optimization.

With regard to the objective function thus formulated, if it is interpreted as a measure of social utility, then social consideration for the individual is neutral, at least within the sphere of the enterprises activities. On the other hand, if it is interpreted as "potential profit" then it inherits the ambiguity of the compensation principle as a criterion for judging social welfare. Either way, this approach has helped to simplify the analysis a great deal. Indivisibility in this case can be dealt with in a much simpler manner. Essentially it involves the requirement that the extra benefit obtained be at least as large as the extra cost. Even in the case of a government project where no market transaction exists (e.g. a public good such as

defense), it makes the job of evaluating the project feasible in the form of cost-benefit analysis. Naturally, one would expect that, in this case, a lot more relevant aspects may not be quantifiable or assumed away and hence a lot more uncertainty is associated with the analysis.

This is probably the reason why, despite Samuelson's view that the concept is "worse than useless", it is still a very popular tool, and with regard to feasibility, sometimes the only tool available. The Samuelson judgement is a little harsh because of his impatience with the fact that there is no hope that the strong assumptions required by the surplus concept can be completely satisfied, especially, the relevance of the concept to the ultimate requirement of the social welfare function.

Most economists would agree that, provided the enterprises activities form only a small part of the total economy and that externalities if they exist are small then the surplus concept does reflect the approximate gain in the context of the approach outlook.

We have seen that Pareto optimal conditions can only be achieved if there is no constraint whatsoever on public enterprises activities. In this case they would generally have to operate on a deficit in view of the fact that most public enterprises activities normally exhibit increasing return (e.e. public utility). To cover the loss, taxation has been looked upon as a possible solution, but unless the tax is of the lump sum type or that every good can be taxed,

its imposition would distort the price system and Pareto conditions can no longer be achieved. Unfortunately, the two types of tax mentioned above are not feasible.

The other alternative is to incorporate the profit constraint in the analysis and hence the pricing policy is formulated in a second best approach. The solution, as expected, would be prices different from marginal costs. This price system would satisfy the constraint and causes the otherwise optimal welfare to be reduced by the least amount possible. Hence the terms "second best" or "quasi optimal" being used to refer to this type of solution.

There are other possible constraints such as capacity, a single price, an upper limit to prices, etc. They arise either due to some peculiarity or practical difficulty related to the activity itself or due to an attempt to translate certain government policy into constraints. With regard to the latter point, it may be worthwhile to study the possibility of formulating government policy into constraints and thus reduce the pressure on specifying the objective function. The alternative is to introduce these aspects into the objective function itself. Felstein (Ref. 24) formulated the population income distribution into the objective function, using the surplus concept, so that the resource allocation would be done with income distribution taken into account. Although what he did is still a very simplistic representation of a social welfare function, the difficulty in interpreting the result is fairly obvious.

In this respect, computer programming would seem to offer much more opportunity as an analytical tool not only because the objective functions and others can be much more easily formulated, not being restricted to the form that would allow calculus analysis possible, but also because of its speed, storage and flexibility that permit the handling of a large number of parameters and variables, and the effect of changes in parameters can be readily obtained and evaluated.

V. CONCLUSION

From parts III and IV, it is seen that subject to some qualifications (no externalities, divisibilities, etc.) and if no profit constraint is imposed then the optimum pricing policy for the public enterprises is indeed operating at the point where equilibrium of supply and demand brings about price equal to marginal cost.

It is seen that this policy is quite neutral toward income distribution. For social welfare functions that require income redistribution to obtain the grand optimum, analyses have been done assuming the lump sum tax for this purpose.

Next, the rigid capacity constraint would result in price deviating from marginal cost. It has been shown that the optimum rigid capacity would be so as those demands constrained by the capacity limit would have to share the capacity cost. The extent of sharing depends on the corresponding demand elasticity. However, if the rigid capacity case is considered as the limit case of a fixed size operation (short-run) then the analyses show that the results could still be considered as prices being equal to marginal cost.

Other constraints such as profit, single price, price limit, etc., when applied unquestionably disturb the price-marginal cost relationship.

It is important to emphasize that the analyses and their conclusions have been made assuming:

- Continuously differentiable indifferent curves,
- Perfect divisibility of plant size and commodities,
- Continuous and known fixed demand and supply curves,
- No joint production (to allow partial derivatives).

In other words, the analyses are static ones with the divisibility assumption only approximately followed if the demand is large compared to plant sizes. The limiting case is in pure competition where the demand is so large (completely elastic) as to become a horizontal line, then indivisibility is no longer a problem.

It can be seen from the underlying assumptions that even if it is possible to operate under no constraint whatsoever, marginal cost pricing would still face a great deal of difficulty in being carried out in practice. The indivisibility of capacity results in the "additional unit" of capacity to be able to bring about more than an "additional unit" of output. Thus if the marginal cost concept is practiced rigidly, the cost of acquiring the "additional unit" of capacity will not be taken into account, only the operating cost would be considered instead. A way of facing this problem is to consider the product produced under different circumstances (when, where, how and even to whom they are produced) as being different products. For example the products produced at peak and off-peak periods are considered different, the fixed cost incurred by the

additional capacity, to satisfy peak demand period only, is considered as the cost involved in producing the product at peak period. Thus it is important to know enough about the cost picture to allow proper cost allocation.

The case of the by-product (joint production) makes it particularly difficult for cost allocation, in the sense that the partial derivative to evaluate marginal cost for product i requires all others to remain constant which cannot be satisfied in this situation. It will therefore be only possible to define the marginal cost for the combined output (x_i, x_j). Other considerations, based on market conditions, that is to say on the demand curves, must be resorted to for individual price determination. For example, consider the simple case of an activity to produce S resulting in two products W and M being available, where $W = k_1 S$ and $M = k_2 S$.

Assume independent demands for W and M with

$$p_1 = p_1(W) = p_1(k_1 S)$$

$$p_2 = p_2(M) = p_2(k_2 S)$$

The cost incurred is $C(S)$.

Then using the surplus concept, the pricing problem would be formulated as:

$$\text{Max.} \int_{0,0}^{k_1 S^*,\, k_2 S^*} (p_1\, dW + p_2\, dM) \quad - \quad C(S^*)$$

The solution would be:

$$k_1 p_1 + k_2 p_2 = \left.\frac{dC}{dS}\right|_{S^*}$$

Thus p_1 and p_2 can only be determined individually with the help of the demand curves.

This case besides, perhaps the hardest thing about evaluating marginal cost is the problem of demand fluctuation with time. It forces the issue of long run planning, of predicting demand behavior which can only be done with some amount of uncertainty, which can be quite considerable, and hence uncertainty about the marginal cost.

Other difficulties related to the determination of marginal cost were pointed out by Dessus (Ref. 25) such as the effect of the time element (e.g., the rate of output production) of the entrepreneurial decisions determining the pattern of the activity and hence the marginal cost. These points are relevant but not as strong as those previously mentioned, in the sense that they do not pose as much of a problem either because the effect may be weak or, as in the case of entrepreneurial decision, once it is made the effect is given and is no longer relevant to the determination of marginal cost.

All this is of course in addition to the externality problem. By this is meant not only the externality on the economy outside the enterprises activities, but also the "externality" due to the way allocation of fixed cost on "different products", say peak demand product. For example

in electric power supply if extra transmission line is used because of heavy load requirement then marginal transmission cost is reduced for all, but the fixed cost of the transmission line would be carried only by peak load demand. This fact results in some form of subsidy.

Further difficulty related to marginal cost pricing is encountered in the development of a tariff structure. As would be expected, the main source of difficulty is in fixed cost allocation.

It has been mentioned previously that fixed cost is dealt with discriminating "the product". Economically, the output of an enterprise can be considered as composed of many different products with different cost functions. Hence the crucial part of marginal cost pricing is in being able to relate to the share in the fixed cost to "the products". Of course the proportional cost (or operating cost) may also change with "different products" but this fact presents no problem due to its unambiguous way of occuring (except for the joint-product situation mentioned above).

This discrimination of "the products", if carried to the detail, would result in an extremely complex tariff structure necessary to determine the prices for an enormous number of products resulting from all possible combinations of time, places, product characteristics, customers. One only needs to think of the cost involved in establishing this tariff structure, assuming it is possible to do so, to realize that the procedure is forbidden. In practice, one

has to look for a way of simplifying this complex task by investigating the possibility of using a limited number of parameters to reflect the occurrence of cost. It is important, in this case, to be able to distinguish between the essential parameters, which will appear explicitly in the tariff, and the secondary parameters whose influence will be lumped together in an averaging process. Boiteux and Stasi (Ref. 26) have stressed the necessity to allow for the special cases where the secondary parameters can have an essential role because of the unusual characteristics involved. Thus to quote them, "...after studying tariffs which are valid for the majority of normal customers, some of the averaging (equalization) will have to be re-examined. Special tariff arrangement will have to be made for exceptional customers."

In the "Tariff Vert", developed for Electricite de France (EDF), the energy has been differentiated based on regions, seasons, times of day and the voltages at which the energy is supplied.

As far as fixed cost is concerned, in the case of EDF as discussed by Boiteux (Ref. 26, 27), it is determined in its relevance to three zones: the collective, semi-individual and individual zones.

1. Capacity in the individual zone is directly determined by the personal peak of the customer and hence the individual should pay the entire charge.

2. In the semi-individual zone, the capacity depends on the uncertain behavior of each individual sharing it. This uncertainty is considered in two aspects:

- The individual variability in consumption,
- The relation of this variability to the collective peak period.

Larger capacity will have to be acquired to support larger variability in consumption, this requirement being more likely the greater if the increase in consumption coincides with the collective peak period. Furthermore, large maximum power demands are relatively less irregular than small demands.

In view of the above observations, and the feasibility reason, it is decided to differentiate the fixed charge according to the level of the contracted power demand, and a degression of the rate of fixed charge as a function of power demand has to be decided upon.

Thus the power contracted for at the peak is invoiced at full rate, and the possible supplements contracted for in the sequence of tariff positions remains (ranked in the order of Full Use Hours in Winter, Full Use Hours in Summer, Slack Hours in Winter, Slack Hours in Summer) at progressively lower rate.

3. In the collective zone, the law of large number takes its effect and averages out individual irregularities. Thus the capacity is essentially determined by the average consumption by the customers at the time of the collective

peak. The related parameter is thus, not the contracted power demand but the energy demand. Thus the fixed cost in this zone is converted to energy cost.

It is also important to consider the shifting effect due to differential energy pricing. It should be so as to flatten the upper level of the demand at the capacity. In the case of EDF, a tariff structure with three hourly positions is capable of providing a fair approximation to the theoretical solution. They are namely: the Peak KWH in Winter, the Full Use KWH in Winter and the Full Use KWH in Summer.

It is thus clearly an effort to construct the true cost function for the "different products". It illustrates the point that fixed costs enter into production functions in different ways as related to the consumer, his pattern of power demand and his pattern of energy consumption. It also shows that the level of detail of cost analysis has to be limited somewhere for feasibility, and at that point an average process (referred to as equalization by French authors) is used to provide the approximation.

We have seen that even if it is possible to price products according to their marginal costs, the practice has a lot of inherent difficulties (dynamic nature, indivisibility, externality, joint product) as well as those due to feasibility considerations (the desirability of a well-stable price system, of a simple and comprehensive tariff structure) which result in the price system being essentially

an approximation of the true one. In reality marginal cost for public enterprises generally incurs a deficit. With regard to this problem, many approaches have been suggested. One is to recover the deficit by collecting taxes but, as has been pointed out, feasible tax measures will result in a distortion of the price system which removes any justification in pricing according to marginal cost. Another approach is to attempt "customer-discrimination" according to their willingness to pay for the subjective value of service, but this would result in misjudgement on the part of the consumers with respect to resource allocation since the prices they are faced with are generally not true social marginal cost. Still another approach is the "second-best" which recognizes the recovery of a deficit as a constraint and finds out how price should deviate from marginal cost so as to diminish the social welfare by the least amount. It is clear that all these approaches result in loss of welfare; which one is better cannot be determined until the welfare function is completely specified.

One argument for the "second best" approach is that it provides a better criterion to guide and evaluate the activity in its capability of self-supporting and allows some degree of autonomy. Furthermore, tax collection generally tends to arouse opposition due to its psychological implication and thus may be politically undesirable.

Other factors that generally cause prices to be different from marginal costs are those considerations which are of

political and social natures. These are essential in any social function and generally result in some group of individuals being subsidized.

In summary, as long as economic efficiency, in the sense that the aggregate production be on the society production forntier, is the only consideration in social welfare, marginal cost pricing will lead to an optimum solution. But this implies the acceptance of the present income distribution. When political and sociological factors are also taken into account, which generally require income redistribution. In this case, if lump sum or all final goods taxes cannot be applied, then the income redistribution can only be done at the expense of economic efficiency and therefore the relative weights of these factors must be considered on the decision of this type.

Even if the condition is favorable toward marginal cost pricing, in practice it is difficult to imagine the marginal cost rule being followed by everyone. This is due to the problems of cost awareness or self-interest and other practical difficulties already discussed. In this case marginal cost pricing is only an attempt to sub-optimize. Within this context, a good pricing policy can only be the result of careful investment planning for the long run so as to be able to deal with the dynamic characteristics of demand and the problem of indivisibility. It is not an easy task and depends very much on each particular activity with its own peculiarity and economic characteristics. Any attempt to

generalize a specific tariff structure for all activities is bound to be either oversimplified or irrelevant in some aspects. The result would be resource misallocation.

It is emphasized again that with respect to marginal cost pricing, the key word is economic efficiency. In the ultimate evaluation of welfare when more than just economic factors are taken into account, marginal cost pricing cannot guarantee an optimum solution unless income redistribution can be carried out in a manner independent of economic consideration by the special types of tax mentioned. Beyond this, nothing can be said about any pricing policy.

As a general rule, however, marginal cost pricing can serve as a useful guidance under circumstances where the question of economic efficiency predominates. When this is not the case, it is still a good idea to construct the marginal cost component, and thus account for the economic efficiency, as far as feasibility allows, while the other component in the price structure is to reflect factors such as politics, government planning policy, sociology and a host of others.

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A great deal of controversy has arisen with regard to the practice of marginal cost pricing as a public sector policy to achieve efficient resource allocation. It is important to present the different aspects of this very complex problem together to establish a more clarified aggregate picture.

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Marginal cost pricing is faced with three main sources of problems:

1. That of measurement, due to both theoretical and practical difficulties in establishing cost,

2. That of determination of social welfare, a very complex task involving not only economic factors but also those of politics, psychology, sociology and hence not easily justified in a clear-cut manner,

3. That of income redistribution, which generally occurs with the achievement of the optimal social welfare in its ultimate form.

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